1 Dyadic Green's Function

- Derive the dyadic Green's function $\overleftrightarrow{G}$ by substituting the scalar Green's function G_0 into the equation below. Discuss the distance dependence $|\vec{r} - \vec{r'}|$.

$$\overleftrightarrow{G}(\vec{r} - \vec{r'}) = \left[\overleftrightarrow{1} + \frac{1}{k^2}\nabla\nabla\right] G_o(\vec{r} - \vec{r'}) \tag{1}$$

- We know how the Fourier spectrum of $\vec{E}$ evolves along the z-axis, as it can be written as

$$\hat{E}(k_x, k_y; z) = \hat{E}(k_x, k_y; 0)e^{\pm ik_z z}, \tag{2}$$

and the inverse Fourier transform can be written as

$$\vec{E}(x, y, z) = \iint_{-\infty}^{+\infty} \hat{E}(k_x, k_y; z)e^{i[k_x x + k_y y]}\mathrm{d}k_x\mathrm{d}k_y. \tag{3}$$

Derive Eq. (2) by inserting the inverse Fourier transform in Eq. (3) into the Helmholtz equation given below. The vector Helmholtz equation is given as

$$(\nabla^2 + k^2)\vec{E}(\vec{r}) = 0. \tag{4}$$

Assume that the Fourier spectrum is known in the plane $z = 0$.

2 Spectral Representation of EM Fields

- The Weyl identity is given by

$$\frac{e^{ik\sqrt{(x^2+y^2+z^2)}}}{\sqrt{(x^2 + y^2 + z^2)}} = \frac{i}{2\pi} \iint_{-\infty}^{+\infty} \frac{e^{ik_x x + ik_y y + ik_z |z|}}{k_z}\mathrm{d}k_x\mathrm{d}k_y. \tag{5}$$

Using the Weyl identity, derive the spatial spectrum $\hat{E}(k_x, k_y; z)$ of an electric dipole at $\vec{r}_0 = (0, 0, z_0)$, with dipole moment $\vec{\mu} = (\mu, 0, 0)$. Consider the asymptotic limit $z \to \infty$ and solve for the electric field $\vec{E}$.