1 Propagation and Focusing of Optical fields

- The reflected image of a laser beam focused on a dielectric interface is given by Eqs. (1)-(3),

$$E(\rho,\phi,z) = E_0 \frac{k_3 f^2}{2f\prime i}\; e^{-ik(z+f\prime)} \sqrt{\frac{n_0}{n_3}} \left[(I_{0r} + I_{2r}\cos 2\phi) n_x - I_{2r}\sin 2\phi n_y\right], \tag{1}$$

$$\begin{aligned} I_{0r}(\rho,z) = \int_0^{\theta_{max}} \mathrm{d}\theta f_\omega(\theta)\cos\theta\sin\theta[r_p(\theta) - r_s(\theta)] J_0\left(k_3\rho\sin\theta\left(\frac{f}{f\prime}\right)\right) \\ \times \exp\left[\frac{i}{2}k_3 z\left[\frac{f}{f\prime}\right]^2\sin^2\theta + 2ik_1 z_0\cos\theta\right], \end{aligned} \tag{2}$$

$$\begin{aligned} I_{2r}(\rho,z) = \int_0^{\theta_{max}} \mathrm{d}\theta f_\omega(\theta)\cos\theta\sin\theta[r_p(\theta) + r_s(\theta)] J_2\left(k_3\rho\sin\theta\left(\frac{f}{f\prime}\right)\right) \\ \times \exp\left[\frac{i}{2}k_3 z\left[\frac{f}{f\prime}\right]^2\sin^2\theta + 2ik_1 z_0\cos\theta\right]. \end{aligned} \tag{3}$$

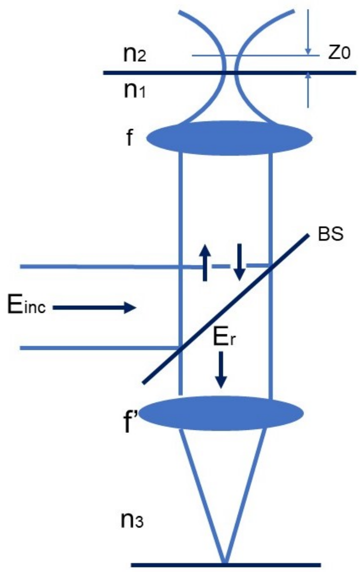

Figure 1: Experimental setup for the investigation of the reflected image of a diffraction limited focused spot. A linearly polarized beam is reflected by a beamsplitter (BS) and focused by a high-NA objective lens with focal radius f onto an interface between two dielectric media n_1 and n_2 [2]. The reflected field is collected by the same lens, transmitted through the beamsplitter and refocused by a second lens with focal radius $f\prime$ in a medium with refractive index n_3.

Derive these equations starting from Eq. (4), which is the collimated reflected field.

$$E_r^{\infty} = -E_{\rm inc} e^{2ik_{z1}Z_0} \Big[[\cos^2\phi r_p(\theta) - \sin^2\phi r_s(\theta)] n_x + \sin\phi\cos\phi [r_p(\theta) + r_s(\theta)] n_y \Big]. \quad (4)$$

Notice that the fields propagate in the negative z-direction.

- The paraxial Gaussian beam is not a rigorous solution of Maxwell's equations. Its field is therefore not divergence free ($\nabla \cdot E \neq 0$). By requiring $\nabla \cdot E = 0$ one can derive an expression for the longitudinal field E_z. Assume that $E_y = 0$ everywhere and derive E_z to the lowest order, for which the solution is non-zero. Sketch the distribution of $|E_z|^2$ in the focal plane.

- Consider the Hermite-Gaussian modes (HG10) defined as

$$\mathbf{E}^H_{nm}(x, y, z) = w_0^{n+m} \frac{\partial^n}{\partial x^n} \frac{\partial^m}{\partial y^m} \mathbf{E}(x, y, z) \quad n = 1,\, m = 0,$$

where

$$\mathbf{E}(x, y, z) = E_0 e^{-\frac{(x^2+y^2)}{w_0^2}} e^{ikz}.$$

Superimpose two HG modes as follows : $HG10\ (\vec{x}) + HG01\ (\vec{y})$

1. What is the polarisation and field profile of the resulting modes?
2. If this mode is focused by a high-NA objective, what would be the expression of the field in the focal region? (derive the diffraction integrals as we did for a Gaussian mode)
3. What is the field polarisation in the focal region?

2 References

1. Principles of Nano-Optics (Second edition) by Lukas Novotny
2. Book chapter : Propagation and focusing of optical fields