Exercise IV

1 Confocal Microscopy

- Consider a sample with a uniform layer of dipolar particles with fixed dipole orientations along the x-axis. The layer is transverse to the optical axis and each element of the layer has a constant polarizability α_{xx} . The sample is illuminated by a focused Gaussian beam and it is translated along the optical axis z. We use both non-confocal (s_1) and confocal (s_2) detection over an area δA. The two signals are well approximated by the Eqs. (1) and (2) given below.

$$\text{non}-\text{confocal}: s_2(x_n, y_n, z_n; \omega) \propto |\alpha_{xx} I_{00}|^2 \delta A. \tag{1}$$

$$\text{confocal}: s_1(x_n, y_n, z_n; \omega) \propto |\alpha_{xx} I_{00}^2|^2 \delta A. \tag{2}$$

(a) Calculate the non-confocal signal as a function of z.
(b) Calculate the confocal signal as a function of z.
(c) What is the conclusion?

Hint: Use the Bessel function closure relations given below.

$$\int_0^\infty J_n(a_1 bx)\ J_n(a_2 bx)\ x\ \mathrm{d}x = \frac{1}{a_1 b^2}\ \delta\ (a_1 - a_2).$$

2 Near-Field Microscopy

- Calculate the difference in transmission through an aluminum-coated aperture probe and an aperture probe with an infinitely conducting coating. Assume an aperture diameter of 100 nm and a taper angle of $\delta = 10°$.
- Apply Babinet's principle to derive the fields near an ideally conducting disk. Use Bethe-Bouwkamp's solution and state the fields in the plane of the disk.

3 References

1. Principles of Nano-Optics (Second edition) by Lukas Novotny