1 Radiation Reaction

The field acting back on the dipole due to radiation is called $\vec{E}_{\text{self}}$ and it can be seen as a **friction**.

$$\vec{F}_R = q\vec{E}_{\text{self}}. \tag{1}$$

The **Abraham Lorentz back action** can be written as

$$\vec{F}_R = \frac{q^2 \dddot{\vec{r}}}{6\pi\epsilon_0 c^3}, \tag{2}$$

where $\vec{\mu} = q\vec{r}$ is the electric dipole moment. Show that the effective polarizability with radiative corrections reads

$$\alpha_{\text{eff}} = \frac{\alpha}{1 - \left[\frac{ik^3}{6\pi\epsilon_0}\alpha\right]},$$

where α is the dipole polarizability.

2 Polarizability of a Classical Point Like Radiator

Calculate the polarizability α_{CL} of a classical point like radiator, with dipole moment $\vec{\mu} = q\vec{r}$, starting from the equation of a forced harmonic oscillator

$$\ddot{\vec{r}} + \Gamma'\dot{\vec{r}} + \tau\dddot{\vec{r}} + \omega_0^2\vec{r} = \frac{e}{m}\vec{E}_0 e^{-i\omega t}, \tag{3}$$

where $\vec{E}_0$ is the applied electric field, Γ' is the non-radiative damping rate, and $\Gamma = \tau\omega_0^2$ is the radiative decay rate from the Abraham Lorentz back action (see previous question).

3 Polarizability of a Two Level System

Calculate the polarizability of a two-level system (TLS) α_{TLS} in the semiclassical theory starting from the Hamiltonian

$$\hat{H} = \vec{H}_{\text{TLS}} - [\hat{\vec{d}} \cdot \vec{E}_0], \tag{4}$$

where $\vec{E}_0$ is the applied electric field, $\hat{\vec{d}}$ is the dipole operator and

$$\vec{H}_{\text{TLS}} = \begin{pmatrix} E_2 & 0 \\ 0 & E_1 \end{pmatrix} = \frac{\hbar\omega_0}{2}\hat{\sigma}_z \tag{5}$$

is the Hamiltonian of the TLS, where $\omega_0 = \frac{E_2 - E_1}{\hbar}$ is the resonance frequency, and $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is the Pauli matrix. Similarly, $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ can be used for the interaction term,

$$\hat{d} \cdot \vec{E} = \begin{pmatrix} 0 & d_{12}\ E_0\ e^{-i\omega t} \\ d_{21}\ E_0\ e^{-i\omega t} & 0 \end{pmatrix} = \hbar V\ e^{-i\omega t}\hat{\sigma}_x, \tag{6}$$

where V is the **Rabi Frequency**, $V = -d_{12}\frac{E_0}{\hbar}$.

4 Coherent/Incoherent Emission of a Two Level System

The polarizability of a TLS is responsible for coherent scattering, which can be calculated as the power scattered by the induced dipole $\vec{\mu} = \alpha_{\mathrm{TLS}}\vec{E}_0$. In the lecture, we also demonstrated that the total power (coherent plus incoherent) emitted by a three-level system is proportional to the excited-state population in the steady state ρ_{22}^{ss} times the radiative decay rate Γ_1

$$P = \hbar\, \omega_0\, \rho_{22}^{ss}\, \Gamma_1. \tag{7}$$

Calculate the coherent and the incoherent power as a function of the incident power and show that at low pump power the power is primarily coherent and that above saturation the power is primarily incoherent.

5 References

1. Principles of Nano-Optics (Second edition) by Lukas Novotny
2. Molecular scattering and fluorescence in strongly confined optical fields by Mario Agio