Exercise VIII

1 Scattering and Absorption Cross-sections of a Metal Nano-particle

Consider a spherical metal nanoparticle (NP) of radius a and dielectric constant ϵ_{NP} in air ($\epsilon_1 = 1$) as an oscillating dipole. Using the expression of the effective polarizability with radiative corrections for the NP, i.e.,

$$\alpha = \frac{\alpha_0}{1 - \imath\frac{k^3}{6\pi}\alpha_0}, \tag{1}$$

where α_0 is the quasistatic polarizability, i.e.,

$$\alpha_0 = 4\pi a^3 \frac{\epsilon_{\mathrm{NP}} - \epsilon_1}{\epsilon_{\mathrm{NP}} + 2\epsilon_1}. \tag{2}$$

- Calculate the scattering and the absorption cross sections of the NP. The cross section (σ) is defined as the scattered and absorbed power ($P_{\mathrm{sca,abs}}$) divided by the incident intensity I_{inc}, i.e., $\sigma_{\mathrm{sca,abs}} = P_{\mathrm{sca,abs}}/I_{\mathrm{inc}}$.
- Show that the sum of the scattered and absorbed power is equal to the extinguished power, i.e, that $\sigma_{\mathrm{ext}} = \sigma_{\mathrm{sca}} + \sigma_{\mathrm{abs}}$, where σ_{ext} is the extinction cross section and the extinguished power P_{ext} is the power removed by the NP in the propagation direction of the incident plane wave.

2 Surface Plasmon-Polariton Waves

- Briefly describe the conditions for the excitation and observation of surface plasmon-polariton (SPP) waves.
- For a medium characterized by a complex frequency-dependent dielectric function $\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)}$, where $\omega_p = 2\pi \times 2.068 \times 10^{15}$ rad/s and $\gamma = 2\pi \times 4.449 \times 10^{12}$ rad/s derive an approximate expression for the propagation length of the SPP wave as a function of ω.