## Problem 1

Determine the Fourier transform of

$$f(t) = \begin{cases} \cos^2 \omega_p t & |t| < T \\ 0 & |t| > T \end{cases}$$

Make a sketch of $F(\omega)$, then sketch its limiting form as $T \to \pm\infty$.

## Problem 2

(a) Show that $F\{1\} = 2\pi\delta(k)$.
(b) Determine the Fourier transform of the function $f(x) = A \cos k_0 x$.

## Problem 3

The rectangular function is often defined as

$$rect \left| \frac{x - x_0}{a} \right| = \begin{cases} 0, & |(x - x_0)/a| > \frac{1}{2} \\ \frac{1}{2}, & |(x - x_0)/a| = \frac{1}{2} \\ 1, & |(x - x_0)/a| < \frac{1}{2} \end{cases}$$

where it is set equal to $\frac{1}{2}$ at the discontinuities (Fig.1). Determine the Fourier transform of

$$f(x) = rect \left| \frac{x - x_0}{a} \right|$$

Notice that this is just a rectangular pulse, like that in Fig. 2, shifted a distance xo from the origin.

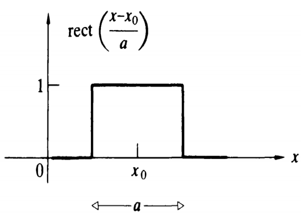


Figure 1: Rectangular function

## Problem 4

Figure 3 shows, in one dimension, the electric field across an illuminated aperture consisting of several opaque bars forming a grating. Considering it to be created by taking the product of a periodic rectangular wave $h(x)$ and a unit rectangular function $f(x)$, sketch the resulting electric field in the Fraunhofer region.

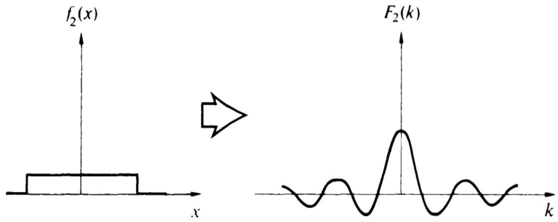


Figure 2: Rectangular pulse

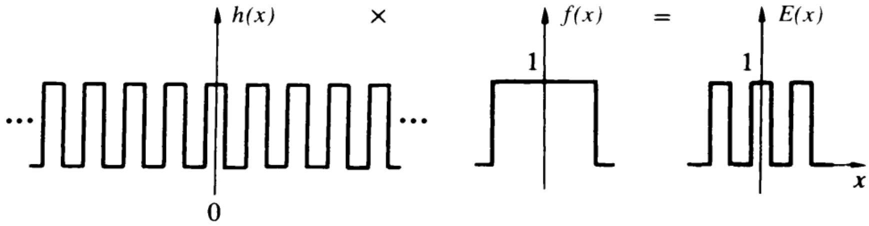


Figure 3: Electric field across an illuminated aperture

## Problem 5

Show that when $f(t) = A\sin(\omega t + \varepsilon)$, $C_{ff}(\tau) = (A^2/2)\cos\omega t2$, which confirms the loss of phase information in the autocorrelation.