Optics: Exercise IV solutions

Problem 1

(a) Given that a P-state light wave has angular frequency ω
Amplitude is E_o
Propagating of the wave is along the x-axis with plane of vibration at an angle of 25◦ to the xy-plane. The disturbance is zero at t = 0 and x = 0.
The expression for this particular light wave can be writter as follows,
$E_{oy} = E_o \cos 25^\circ$
$E_{oz} = E_o \sin 25^\circ$

$E(x,t) = E_y + E_z$
$= E_o \cos 25^\circ \sin(kx-\omega t) + E_o \sin 25^\circ \sin(kx-\omega t)$
$E(x,t) = (0.91\hat{j} + 0.42\hat{k}) E_o \sin(kx-\omega t)$
$E(x,t) = (0.91\hat{j} + 0.42\hat{k}) E_o \cos(kx-\omega t-\pi 2)$

(b) The intensity of natural light will decrease to half of the incident intensity when it passes through a linear polarizer, but the incident intensity will not change by λ/4 plate (quarter wave plate).

(c) Refractive index of glass is given as n_g= 1.65, Refractive index of ethyl alcohol is given as n_g= 1.36. Light reflected from a glass plate immersed in ethyl alcohol is found to be completely linearly polarized.

The angle at which the partially polarized beam be transmitted into the plate can be found using Maxwell's equations.

$$\left(E_{0\|}\right)_{ref} = \left(E_{0\|}\right)_{inc} \frac{n_1 \cos\theta_2 - n_2 \cos\theta_1}{n_1 \cos\theta_2 + n_2 \cos\theta_1}$$
$$\left(E_{0\|}\right)_{ref} = \left(E_{0\|}\right)_{inc} \frac{\tan(\theta_1-\theta_2)}{\tan(\theta_1+\theta_2)}$$

Now if $\theta_1 + \theta_2 = 90^\circ$ the parallel component of reflection becomes zero. Then in this case we know:

$\sin\theta_2 = \sin(90-\theta_1) = \cos\theta_1$

then:

$n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow n_1 \sin\theta_1 = n_2 \cos\theta_1$

$\Rightarrow \tan\theta_1 = \frac{n_2}{n_1}$

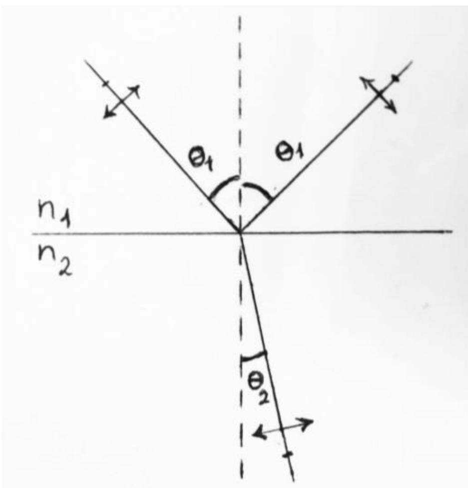
θ_1 is Brewster angle

and when $n_1 = 1.36$, $n_2 = 1.65$

: $\tan\theta_1 = \frac{1.65}{1.36} = 1.21$ and $\theta_1 = 50.5^\circ$

$n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow \sin\theta_2 = \frac{n_1}{n_2} \sin\theta_1$

$\sin\theta_2 = \frac{1.36}{1.65} \sin(50.5^\circ) = 39.5^\circ$



Problem 2

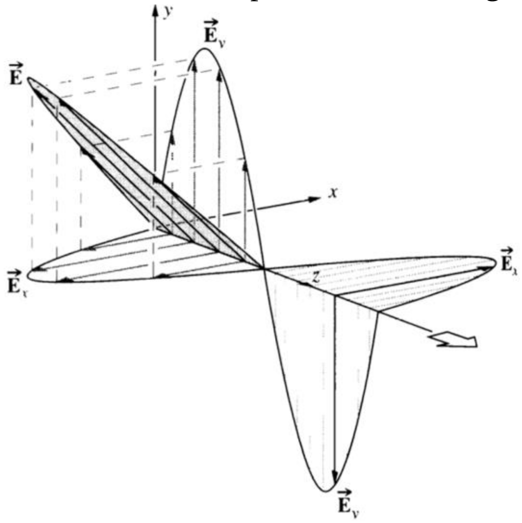
In each part the x and y components have the same amplitude E_0 as shown in the picture.

a) $E = (\hat{i} - \hat{j})\, E_0 \cos(kz-\omega t)$. E_y leads E_x by π and as a result the P-state is linearly polarized in a line $3\pi/4$ and $-\pi/4$.

b) As we known: $k = 2\pi/\lambda$ and $\omega = 2\pi\nu$, $\vec{E} = (\hat{i} - \hat{j})\, E_0 \sin(kz-\omega t)$ and again E_y leads E_x by π and the P-state is the same as part a)

c) E_x leads E_y by $\pi/4$ Therefore it is an elliptical polarization and left handed.

d) E_y leads E_x by $\pi/2$. Therefore, it is circular polarization and right handed.



Problem 3

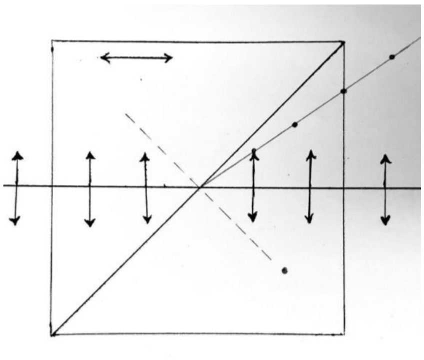
The un polarized light will become R-Circular polarized when it passes through a polarizer and $\lambda/4$. But when the light is reflected by a glass surface, the polarization of the light changes to L-Circular and therefore is totally absorbed by R-Circular system. It is also possible to show this with Muller matrix:

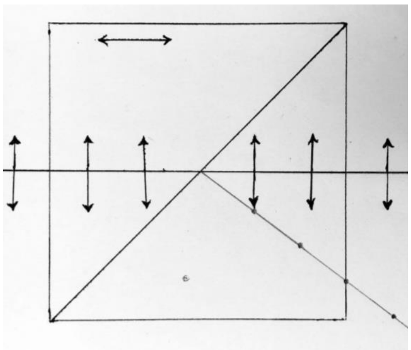
$$\underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\substack{Quarter-wave\ plate \\ fast\ axis\ vertical}} \cdot \frac{1}{2} \underbrace{\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}}_{\substack{linear\quad polarizer \\ at\quad +45^\circ}} \cdot \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{Unpolarized} = \frac{1}{2} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

This shows R-state polarized with half intensity. Then after reflection we have

$$\frac{1}{2} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \cdot \frac{1}{2} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Problem 4


Calcite


Quartz

1 and 2.

At λ = 580 nm

Calcite: $n_0 = 1.65$, $n_e = 1.48$

Quartz: $n_0 = 1.544$, $n_e = 1.553$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

In Rochon BS the optical axis is for the first prism in the direction of light propagation and the optical axis of second prism is only perpendicular to one polarization. Therefore, for the ordinary beam the refractive index will not change.

3. The dichroic polarizer absorbs one polarization and this could generate heat in the system but the Rochon BS only separate the beam depends on the polarization.

4. in Wollaston since each prism i orthogonal to one polarization, therefore both polarized beam is deviated and this can make a mis-alignment in system.