Optics Exercise V Solutions

2019

Problem 1

$(R + L)^2 = R^2 + a^2$

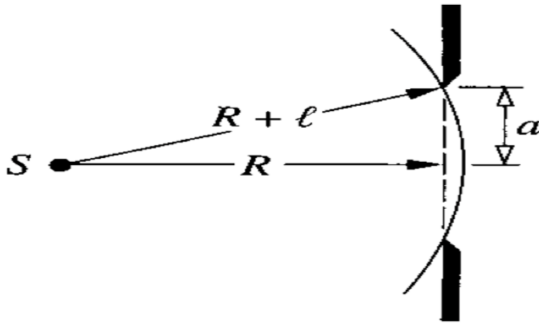
$R^2 + L^2 + 2RL = R^2 + a^2$

$L^2 + 2RL = a^2$

$R = (a^2 - L^2) / 2L$

$R \approx a^2 / 2L$

$LR = a^2 / 2$



for $\lambda \gg L \rightarrow \lambda R \gg \frac{a^2}{2}$

$(\lambda/10)\, R = \{(1\times10^{-3})^2\}/ 2$

$\Rightarrow R = \{(1\times10^{-3})^2\}/ 2\ \{10 / (500\times10^{-9})\}$

$\Rightarrow R = 10$ m

Problem 2

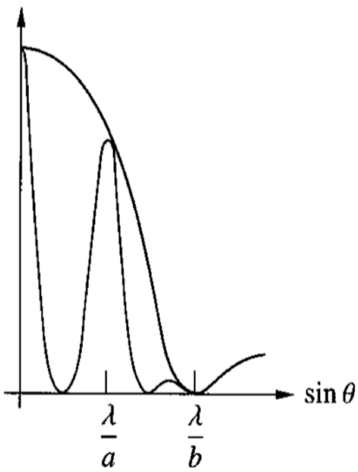
$\beta = \frac{kb}{2} \sin\theta$

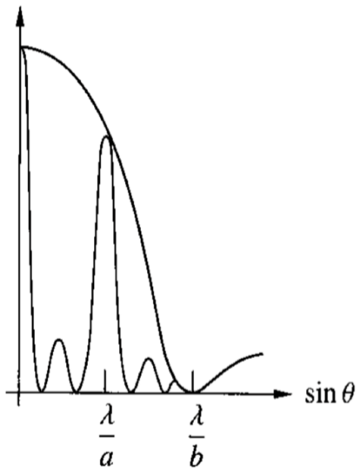
$\alpha = \frac{ka}{2} \sin\theta$

$I(\theta) = I_0 \{ \frac{\sin\beta}{\beta} \}^2 \{ \frac{\sin N\alpha}{\sin\alpha} \}^2$

for N = 1 → single slit

for N = 2 → double slit





if $\theta = 0 \rightarrow I(0) = N^2 I_0$

$\frac{I_{(\theta)}}{I_{(0)}} = \frac{1}{N^2} \{ \frac{\sin\beta}{\beta} \}^2 \{ \frac{\sin N\alpha}{\sin\alpha} \}^2$

$\frac{\sin N\alpha}{\sin\alpha} = N$ is the principle maxima $\rightarrow \alpha = 0, \pi, 2\pi$

$\frac{\sin N\alpha}{\sin\alpha} = 0$ is the principle minima $\rightarrow \alpha = \pm \frac{\pi}{N}, \pm \frac{2\pi}{N}, \pm \frac{3\pi}{N}$ normally the maxima is between two minima then

$\alpha = \pm \frac{3\pi}{2N}, \pm \frac{5\pi}{2N}$

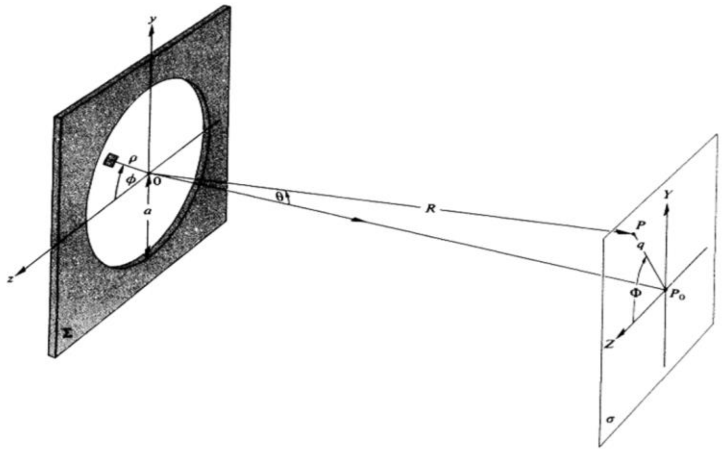
if $N = 3 \rightarrow \alpha = \frac{\pi}{2}$

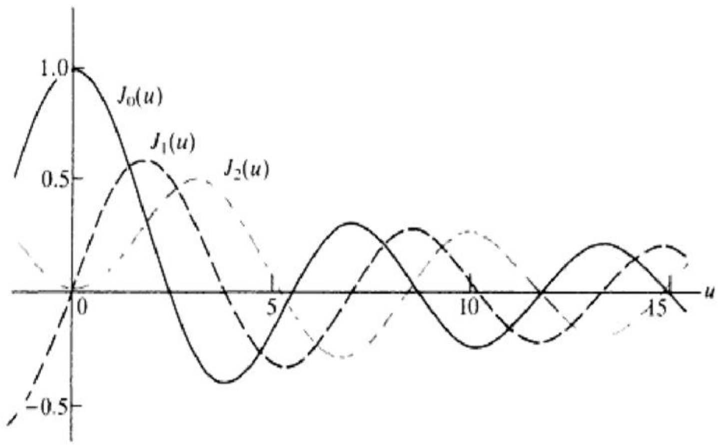
$\frac{I_{(\theta)}}{I_{(0)}} = \frac{1}{9} \{ \frac{\sin\beta}{\beta} \}^2$

Problem 3

$$I(\theta) = I_0\left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right]^2$$

$$\text{Bessel function}: \quad J_m(u) = \frac{i^{-m}}{2\pi}\int_0^{2\pi} e^{i(mv+u\cos v)}dv$$





Where m is the order of Bessel function.

Here, $\sin\theta = \frac{q}{R}$.

The central maximum corresponds to a high irradiance circular spot known as the Ariy-disk. To find the radius of the Ariy-disk, we should consider the dark minimum after the bright disk in center.

In this case, when $J_1(ka\sin\theta) = 0$. we have the first minimum.

By checking the value in Bessel-function table for $J_1(u) = 0$, then we have u = 3.83.

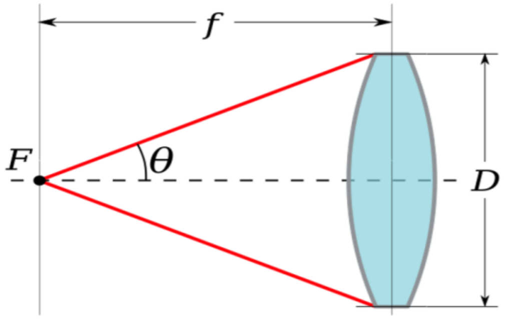
This means: $\frac{kaq}{R} = 3.83 \rightarrow \frac{3.83R\lambda}{2\pi a} \rightarrow q = \frac{1.22R\lambda}{2a}$

Regarding the first ex., the $\lambda R \gg \frac{a^2}{2}$ to see the diffraction pattern therefore $a^2 \ll 2\lambda R$ for a lens which is focused on a screen the value $R \approx f$.

We have $q = \frac{1.22 f\lambda}{D}$

where D is the lens diameter and q is the Ariy disk radius.

For objective lens:



$NA = n\sin\theta = n \sin[\arctan(\frac{D}{2f})] = n\frac{D}{2f}$

$f/D = N \approx 1/2NA$

$q = \frac{1.22\lambda}{2NA}$

Problem 4

$\frac{\sin N\alpha}{\sin\alpha} = N \rightarrow \alpha = 0, \pm\pi, \pm 2\pi$

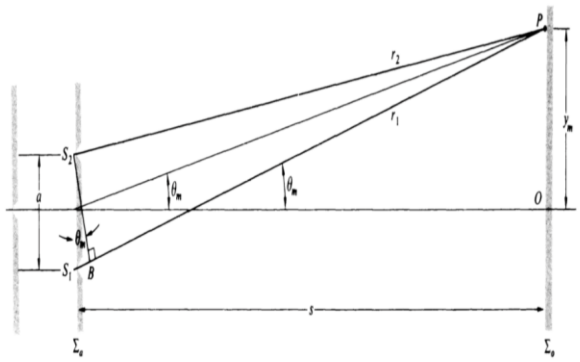
$\alpha = (k\frac{2}{a}) \sin\theta \rightarrow a \sin\theta_m = m\lambda$

$\rightarrow 6\times10^{-6} \sin\theta_3 = 3\times5\times10^{-7} \rightarrow \sin\theta_3 = 0.25 \rightarrow \theta_3 = 14.48$

Problem 5

$\tan\theta_m = \sin\theta_m = \theta_m$

from grating equation



$\tan\theta_m = \sin\theta_m = \theta_m$

$a \sin\theta_m = m\lambda \rightarrow a\frac{Z_m}{R} = m\lambda$

10000 lines/cm $= 10^6$ lines/m

so $a = 10^{-6}$ m

$z_1 (589.9923) = \frac{1\times589.5923\times10^{-9}}{10^{-6}}\times1 = 0.5895$ m

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$\rightarrow z_1 (589.9923) - z_1 (588.9953) = 5.97\times10^{-4}$ m

Problem 6

FSR is the spacing in optical frequency between 2 successive reflected or transmitted optical density maxima or minima of diffractive optical element. FSR is the largest wavelength range for a given order that does not overlap the same range in an adjacent order. If the (m + 1)th order of λ and mth order of (λ + Δλ) lie at the same angle then

$FSR = \frac{\lambda}{m}$

Finesse = $\frac{FSR}{\lambda}$

Q = $\frac{\lambda_0}{\Delta\lambda} = \frac{\gamma_0}{\Delta\gamma_0}$

chromatic resolving power : R = $\frac{\lambda}{\Delta\lambda_{min}}$= mN.

and a ($\sin\theta_m - \sin\theta_i$) = mλ → R = $\frac{N_a\,[\sin\theta m - \sin\theta_i]}{\lambda}$

if two lines with λ and (λ + Δλ) in successive orders (m + 1) and m just coincide

then a ($\sin\theta_m - \sin\theta_i$) = (m + 1)λ = m(λ + Δλ) → ($\Delta\lambda_{FSR}$) = $\frac{\lambda}{m}$

R = m. N → 10^6 = m (260×300) → m =$\frac{10^3}{78}$

FSR = $\frac{\lambda}{m} = \frac{500\times10^{-9}}{\frac{10^3}{78}}$ = 39×10^{-9}

In Fabry-Perot:

R =finesse= F .m

And, mλ = 2d

R = $\frac{F2d}{\lambda} = \frac{25\times2\times0.01}{500\times10^{-9}}$ = 10^6