Problem 1

The intensity of natural light will decrease to half of the incident intensity when it passes through a polarizer, but the incident intensity will not change by $\lambda/4$.

Problem 2

The unpolarized light will become R-Circular polarized when it passes through a polarizer and $\lambda/4$. But when the light is reflected by a glass surface, the polarization of the light changes to L-Circular and therefore is totally absorbed by R-Circular system. It is also possible to show this with Muller matrix:

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\substack{Quarter-wave\ plate \\ fast\ axis\ vertical}} \cdot \frac{1}{2} \underbrace{\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}}_{\substack{linear\ \ polarizer \\ at\ \ +45^\circ}} \cdot \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{Unpolarized} = \frac{1}{2} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

This shows R-state polarized with half intensity.
Then after reflection we have

$$\frac{1}{2} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \cdot \frac{1}{2} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Problem 3

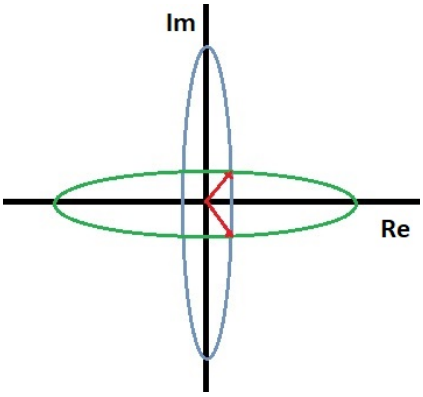
$$\vec{E}_1 = \begin{bmatrix} 1 \\ -2i \end{bmatrix}$$

$$\vec{E}_1 . \vec{E}_2 = \begin{bmatrix} 1 \\ -2i \end{bmatrix} \cdot \begin{bmatrix} E_{11} \\ E_{12} \end{bmatrix} = 0$$

$$\Rightarrow \quad E_{12} = 1, \quad E_{11} = 2i$$

$\lambda/4$ with fast axis at $\theta = e^{-i\pi/4} \begin{pmatrix} \cos^2\theta + i\sin^2\theta & (1-i)\sin\theta\cos\theta \\ (1-i)\sin\theta\cos\theta & \sin^2\theta + i\cos^2\theta \end{pmatrix}$

$\lambda/2$ with fast axis at $\theta = e^{-i\pi/2} \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$



Problem 4

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Problem 5

A general linear retarder with its fast axis parallel to the x-axis of Cartesian.

$$M_R = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\delta & \sin\delta \\ 0 & 0 & -\sin\delta & \cos\delta \end{pmatrix}$$

if the linear retarder has been rotated an angle α with respect to the x-axis then

$$M^r = R(-\alpha)\, M_R R(\alpha)$$

where

$$R(\alpha) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\alpha & \sin 2\alpha & 0 \\ 0 & -\sin 2\alpha & \cos 2\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow \quad M^r \;= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\alpha + \cos\delta \sin^2 2\alpha & (1-\cos\delta)\cos 2\alpha \sin 2\alpha & -\sin\delta \sin 2\alpha \\ 0 & (1-\cos\delta)\cos^2\alpha \sin 2\alpha & \cos\delta \cos^2 2\alpha + \sin^2 2\alpha & \cos 2\alpha \sin\delta \\ 0 & \sin\delta \sin 2\alpha & -\cos 2\alpha \sin\delta & \cos\delta \end{pmatrix}$$

For $\lambda/4$ the δ is $\pi/2$ (This is obvious if we assume $\lambda = 2\pi$)
For $\lambda/2$ th $\delta = \pi$
now if we put $\delta = \pi/2$ and $\alpha = 45°$ then we have

$$M^r = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

when we shine linear light polarizes at 45°

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

when we shine horizontal p-state light

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

The horizontal p-polarized converts to right handed circular light (R-stat).