Problem 1

$$E_1 = E_0 \exp\left[i\tfrac{2\pi}{\lambda}z\right]$$
$$E_2 = E_0 \exp\left[i\tfrac{2\pi}{\lambda}\left(z\cos\theta + y\sin\theta\right)\right]$$

In xy plane $z = 0$

$$\Rightarrow E = E_1 + E_2 = E_0\left[1 + \exp\left[i\frac{2\pi}{\lambda}\left(y\sin\theta\right)\right]\right]$$

We know :

$$|1 + e^{ix}|^2 = \sin^2 x + \cos^2 x + 2\cos x + 1 = 2\cos x + 2$$

On the other hand:

$$|E|^2 = I \Rightarrow$$
$$I\left(y\right) = 2{E_0}^2\left[1 + \cos\left(\frac{2\pi}{\lambda}y\sin\theta\right)\right]$$

In trigonometric relations:

$$\cos 2\theta = 2\cos^2\theta - 1$$
$$\Rightarrow I\left(y\right) = \left[4{E_0}^2\cos^2\left(\frac{\pi}{\lambda}y\sin\theta\right)\right]$$

$$I\left(y\right) = 0$$

When

$$y\frac{\pi}{\lambda}\sin\theta = \left(2m + 1\right)\frac{\pi}{2}$$
$$\Rightarrow y = \left(2m + 1\right)\frac{\lambda}{2\sin\theta}$$
$$y_2 - y_1 = \left(2\left(m + 1\right) + 1 - 2m + 1\right)\frac{\lambda}{2\sin\theta} = 2\frac{\lambda}{2\sin\theta} = \frac{\lambda}{\sin\theta}$$
$$\delta = k\left(r_1 - r_2\right) + \left(E_1 - E_2\right)$$

and

$$I = 2I_0\left(1 + \cos\delta\right) = 4I_0\cos^2\frac{\delta}{2}$$

This is true when separation between two sources is small in comparison with r_1 and r_2. In this condition $\vec{E_{01}}$ and $\vec{E_{02}}$ may be considered independent of position.

Problem 2

(a) According to the picture, there is a dark fringe for:

$$r_2 - r_1 = \pm\frac{\lambda}{2}$$

if a is the distance between two slits, then:

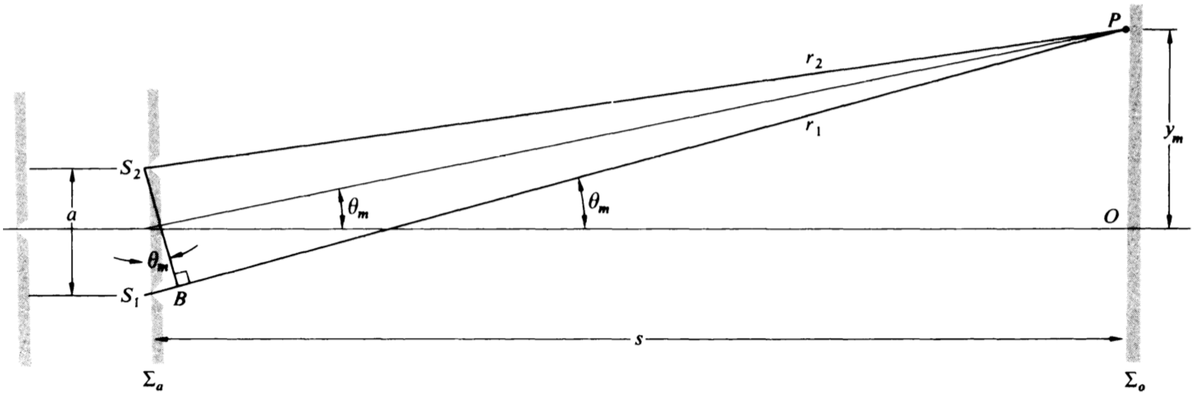
$$a\sin\theta_m = \pm\tfrac{\lambda}{2} \;\Rightarrow \theta_m \approx \pm\tfrac{\lambda}{2a} = \tfrac{632.8\times10^{-9}m}{2\times0.2\times10^{-3}m} = 1.58\times10^{-3}$$
$$y_m = s\theta_m = \pm\left(1\times1.58\times10^{-3}\right) = \pm1.58\ mm$$

(b) For bright fringes:

$$y_m = sm\tfrac{\lambda}{a}$$
$$y_5 = s5\tfrac{\lambda}{a} = 1.58\times10^{-2}\ m = 15.8\ mm$$

(c) Since the fringes vary as $\cos^2(\theta)$

Problem 3



In constructive condition; $OPD = m\lambda$ Here since the impinging light has an angle it will not reach the holes at the same path and has a $a\sin(\theta)$ path difference. Therefore:

$$r_1 - r_2 + a\sin\theta = m\lambda$$
$$\theta_m \simeq \frac{y}{s}$$
$$r_1 - r_2 = a\frac{y}{s}$$
$$\Rightarrow r_1 - r_2 = m\lambda - a\sin\theta = a\left(\frac{y}{s}\right) = a\theta_m$$
$$\Rightarrow \theta_m = \left(m\frac{\lambda}{a}\right) - \sin\theta$$

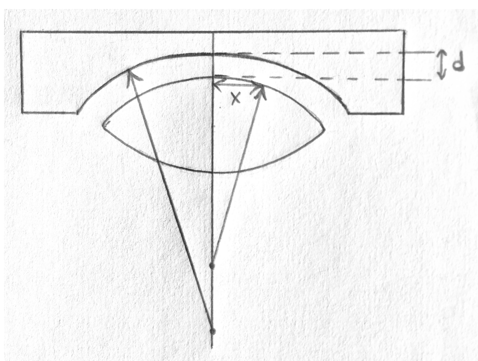
Problem 4

For a region of the layer in vertical reflection, $\theta = 90$ and $m = 0$

$$\lambda_f = \frac{\lambda_0}{n} = \frac{633}{1.34} = 472.3\ nm$$

$$d\cos\theta_t = (2m+1)\frac{\lambda_f}{4} \Rightarrow d\,(1) = (2\times 0+1)\frac{472.3}{4} = 118\ nm$$

Problem 5



$$x^2 = R^2 - (R-d)^2 = R^2 - \left(R^2 + d^2 - 2Rd\right)$$
$$x^2 = d_1\left(R_1 - d_1 + R\right) = 2R_1 d_1 - {d_1}^2$$

Similarly:

$$x^2 = 2R_2 d_2 - {d_2}^2$$
$$d = d_1 - d_2 = \tfrac{x^2}{2}\left[\tfrac{1}{R_1} - \tfrac{1}{R_2}\right] = \tfrac{x^2(R_2 - R_1)}{2R_1R_2}$$

We have minima if:

$$d\cos\theta = 2m\frac{\lambda_f}{4}$$

In this case we have:

$$\tfrac{m\lambda_f}{2} = {x_m}^2\tfrac{(R_2-R_1)}{2R_1R_2} \Rightarrow \sqrt{\tfrac{m\lambda_f R_1 R_2}{R_2 - R_1}} = x_m$$
$$\lim_{R_2\to\infty}\left[R_1R_2m\tfrac{\lambda_f}{(R_2-R_1)}\right]^{\frac{1}{2}} = \left(R_1 m\lambda_f\right)^{\frac{1}{2}}$$