Problem 1

$$(R+L)^2 = R^2 + a^2$$

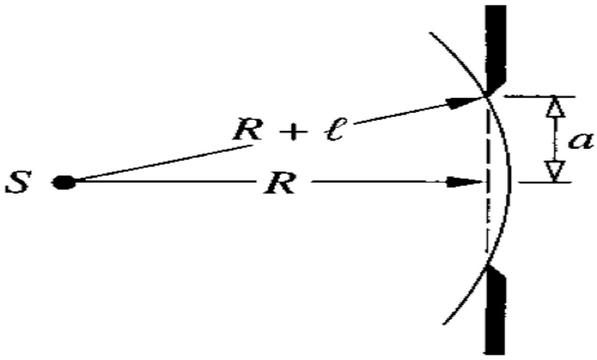
$$R^2 + L^2 + 2RL = R^2 + a^2$$

$$L^2 + 2RL = a^2$$

$$R = \frac{a^2 - L^2}{2L}$$

$$R \approx \frac{a^2}{2L}$$

$$LR = \frac{a^2}{2}$$



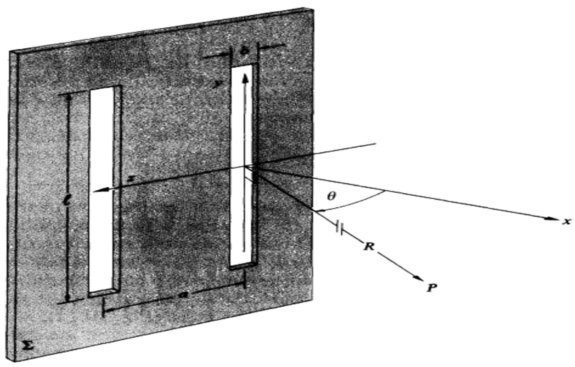
$$for \quad \lambda \gg L \quad \rightarrow \quad \lambda R \gg \frac{a^2}{2}$$

$$\left(\frac{\lambda}{10}\right) R = \frac{\left(1\times10^{-3}\right)^2}{2}$$

$$\Rightarrow R = \frac{\left(1\times10^{-3}\right)^2}{2}\left(\frac{10}{500\times10^{-9}}\right)$$

$$\Rightarrow R = 10\ m$$

Problem 2



$$\beta = \left(\frac{kb}{2}\right)\sin\theta$$

$$\alpha = \left(\frac{ka}{2}\right)\sin\theta$$

$$I\left(\theta\right) = I_0\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2$$

$$for \quad N = 1 \rightarrow single\ slit$$

$$for \quad N = 2 \rightarrow double\ slit$$

$$if \quad \theta = 0 \rightarrow I(0) = N^2 I_0$$

$$\frac{I\left(\theta\right)}{I\left(0\right)} = \frac{1}{N^2}\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2$$

$\frac{\sin N\alpha}{\sin\alpha} = N$ is the principle maxima $\rightarrow \quad \alpha = 0, \pi, 2\pi$

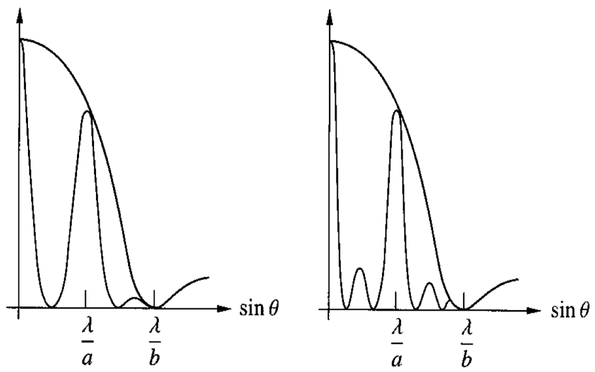
$\frac{\sin N\alpha}{\sin\alpha} = 0$ is the principle minima $\rightarrow \quad \alpha = \pm\frac{\pi}{N}, \pm\frac{2\pi}{N}, \pm\frac{3\pi}{N}$

normally the maxima is between two minima then

$$\alpha = \pm\frac{3\pi}{2N}, \pm\frac{5\pi}{2N}$$

$$if \quad N = 3 \quad \rightarrow \quad \alpha = \frac{\pi}{2}$$

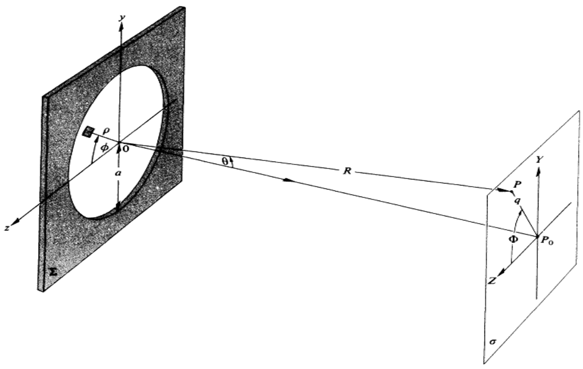
$$\frac{I\left(\theta\right)}{I\left(0\right)} = \frac{1}{9}\left(\frac{\sin\beta}{\beta}\right)^2$$

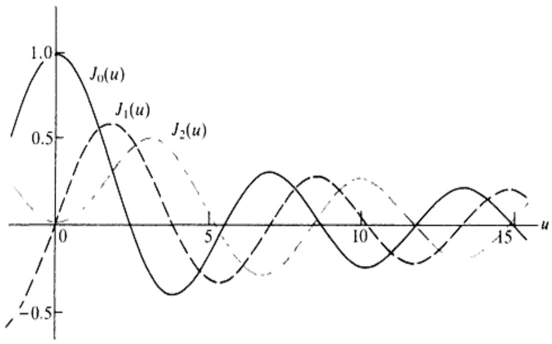


Problem 3

$$I\left(\theta\right) = I_0\left[\frac{2J_1\left(ka\sin\theta\right)}{ka\sin\theta}\right]^2$$

$$\text{Bessel function}: \quad J_m\left(u\right) = \frac{i^{-m}}{2\pi}\int_0^{2\pi} e^{i(mv+u\cos v)}dv$$





Where m is the order of Bessel function.
Here, $sin\theta = \frac{q}{R}$.
The central maximum corresponds to a high irradiance circular spot known as the Ariy-disk. To find the radius of the Ariy-disk, we should consider the dark minimum after the bright disk in center. In this case, when $J_1(kasin\theta) = 0$ we have the first minimum.
By checking the value in Bessel-function table for $J_1(u) = 0$ then we have $u = 3.83$. This means:

$$\frac{kaq}{R} = 3.83 \rightarrow \frac{3.83R\lambda}{2\pi a} \rightarrow q = 1.22\frac{R\lambda}{2a}$$

Regarding the first ex., the $\lambda R \gg \frac{a^2}{2}$ to see the diffraction pattern therefore $a^2 \ll 2\lambda R$ for a lens which is focused on a screen the value $R \approx f$ and we have

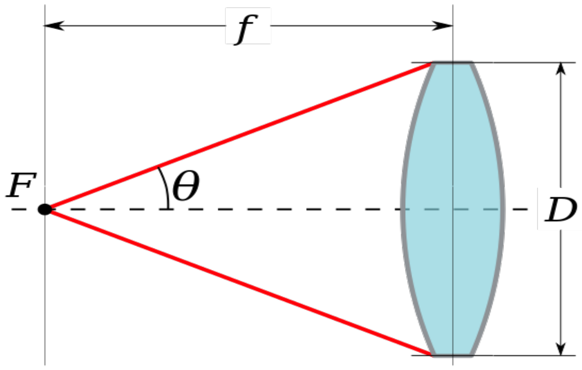
$$q = 1.22\frac{f\lambda}{D}$$

where D is the lens diameter and q is the Ariy disk radius.

For objective lens:

$$NA = n\sin\theta = n\sin\left[\arctan\left(\frac{D}{2f}\right)\right] = n\frac{D}{2f}$$
$$\frac{f}{D} = N \approx \frac{1}{2NA}$$

$$q = 1.22\frac{\lambda}{2NA}$$



Problem 4

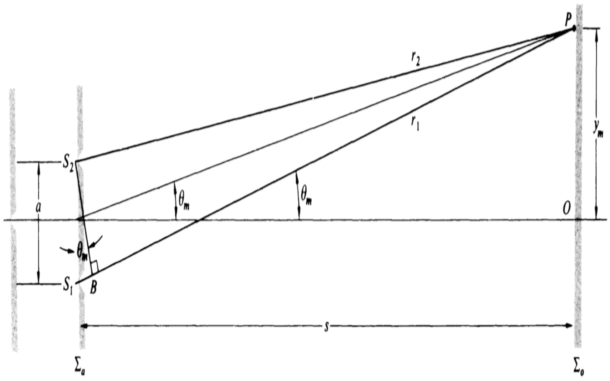
$$\frac{\sin N\alpha}{\sin\alpha} = N \quad \rightarrow \quad \alpha = 0, \pm\pi, \pm 2\pi$$

$$\alpha = \left(k\frac{2}{a}\right)\sin\theta \quad \rightarrow \quad a\sin\theta_m = m\lambda$$

$$\rightarrow \quad 6\times 10^{-6}\sin\theta_3 = 3\times 5\times 10^{-7}$$

$$\rightarrow \quad \sin\theta_3 = 0.25 \quad \rightarrow \quad \theta_3 = 14.48$$

Problem 5



$$\tan\theta_m = \sin\theta_m = \theta_m$$

from grating equation

$$a\sin\theta_m = m\lambda \quad \rightarrow \quad a\frac{z_m}{R} \;= m\lambda$$

$$10000\, lines/cm = 10^6\, lines/m \quad so \quad a = 10^{-6}\, m$$

$$z_1\,(589.9923) = \frac{1\times 589.5923\times 10^{-9}}{10^{-6}}\times 1 = 0.5895\, m$$

$$z_1\,(588.9953) = \frac{1\times 588.9923\times 10^{-9}}{10^{-6}}\times 1 = 0.588995\, m$$

$$\rightarrow \quad z_1\,(589.9923) - z_1\,(588.9953) = 5.97\times 10^{-4}\, m$$

Problem 6

FSR is the spacing in optical frequency between 2 successive reflected or transmitted optical density maxima or minima of diffractive optical element. FSR is the largest wavelength range for a given order that does not overlap the same range in an adjacent order. If the $(m+1)th$ order of λ and mth order of $(\lambda + \Delta\lambda)$ lie at the same angle then

$$FSR = \frac{\lambda}{m}$$

$$Finesse = \frac{FSR}{\lambda}$$

$$Q = \frac{\lambda_0}{\Delta\lambda} = \frac{\gamma_0}{\Delta\gamma_0}$$

$$\text{chromatic resolving power :} \quad R = \frac{\lambda}{(\Delta\lambda)_{\min}} = mN.$$

$$and \;\; a(\sin\theta_m - \sin\theta_i) = m\lambda \;\rightarrow\; R = \frac{Na(\sin\theta_m - \sin\theta_i)}{\lambda}$$

if two lines with λ and $(\lambda + \Delta\lambda)$ in successive orders $(m+1)$ and m just coincide then

$$a(\sin\theta_m - \sin\theta_i) = (m+1)\lambda = m(\lambda + \Delta\lambda)$$

$$\rightarrow \quad (\Delta\lambda_{FSR}) = \tfrac{\lambda}{m}$$

$$R = m.N \rightarrow 10^6 = m\,(260 \times 300) \rightarrow m = \tfrac{10^3}{78}$$

$$FSR = \frac{\lambda}{m} = \frac{500 \times 10^{-9}}{\frac{10^3}{78}} = 39 \times 10^{-9}$$

In Fabry-Perot:

$$R = \overbrace{F}^{finesse}.m \qquad and \quad m\lambda = 2d$$

$$R = \tfrac{F2d}{\lambda} = \tfrac{25 \times 2 \times 0.01}{500 \times 10^9} = 10^6$$

Problem 7

like Problem 4 for $m = 1$ we have

$$a\tfrac{z_m}{R} = m\lambda \quad \rightarrow \quad a = \tfrac{R\lambda}{z_m} \;\; = \frac{12 \times 10^{-6}}{12 \times 10^{-2}} = 10^{-4}$$