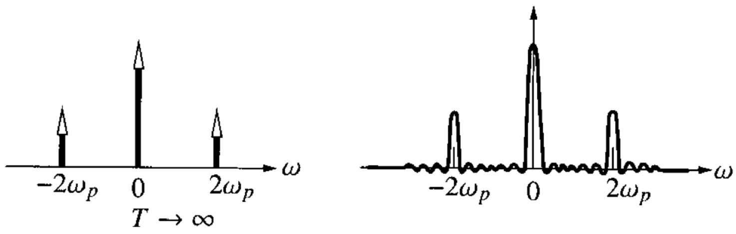
1 Aufgabe 11.3

$$\cos^2 \omega_p t = \frac{1}{2} + \frac{1}{2}\cos 2\omega_p t = \frac{1}{2} + \frac{e^{2i\omega_p t} + e^{-2i\omega_p t}}{4}$$

$$F(\omega) = \frac{1}{2}\int_{-T}^{T} e^{i\omega t}dt + \frac{1}{4}\int_{-T}^{T} e^{i(\omega+2\omega_p)t}dt + \frac{1}{4}\int_{-T}^{T} e^{i(\omega-2\omega_p)t}dt$$

$$F(\omega) = \frac{1}{\omega}\sin(\omega T) + \frac{1}{2(\omega+2\omega_p)}\sin(\omega T + 2\omega_p T) + \frac{1}{2(\omega-2\omega_p)}\sin(\omega T - 2\omega_p T)$$

$$F(\omega) = T sinc(\omega T) + \frac{T}{2} sinc(\omega T + 2\omega_p T) + \frac{T}{2} sinc(\omega T - 2\omega_p T)$$



2 Aufgabe 11.19

Allgemein haben wir

$$g(X) = f(x) \otimes h(x) = \int_{-\infty}^{\infty} f(x)h(X-x)dx$$

und hier

$$f(x-x_0) \otimes h(x) = \int_{-\infty}^{\infty} f(x-x_0)h(X-x)dx$$

Wir setzen $x - x_0 = \alpha$ und erhallten

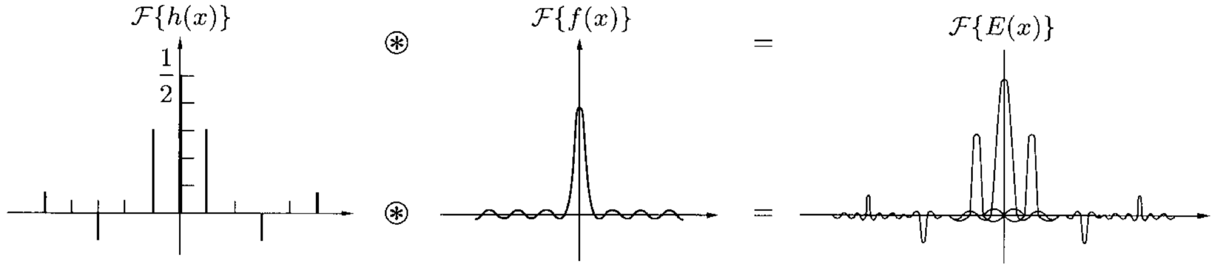
$$\int_{-\infty}^{\infty} f(\alpha)h(X-\alpha-x_0)d\alpha = g(X-x_0$$

3 Aufgabe 11.28

4 Aufgabe 11.30

$$E(Y,Z) = \iint_{-\infty}^{\infty} A(y,z)e^{ik(Yy+Zz)/R}\,dy\ \ dz$$

$$E'(Y,Z) = \iint_{-\infty}^{\infty} A(\alpha y,\beta z)e^{ik(Yy+Zz)/R}\,dy\ \ dz;$$



wir setzen nun $y' = \alpha y$ und $z' = \beta z$ und so ist

$$E'(Y,Z) = \frac{1}{\alpha\beta} \iint_{-\infty}^{\infty} A(y',z') e^{ik\,[(Y/a)y' + (Z/\beta)z']}\, dy'\, dz'$$

$$oder\;\; E'(Y,Z) = \frac{1}{\alpha\beta} E\left(\frac{Y}{\alpha}, \frac{Z}{\beta}\right).$$

5 Aufgabe 11.31

$$\begin{aligned} C_{ff}(\tau) &= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} f(t)\, f(t-\tau)\, dt \\ &= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} A\sin(\omega t + \epsilon)\, A\sin(\omega t - \omega\tau + \epsilon) dt \end{aligned}$$

Wir wissen

$$\cos\alpha - \cos\beta = -2\sin(\frac{\alpha+\beta}{2})\ \sin(\frac{\alpha-\beta}{2})$$

Wann $\alpha = \omega\tau$ und $\beta = 2\omega t - \omega\tau + 2\epsilon$ dann:

$$\begin{aligned} C_{ff}(\tau) &= \lim_{T\to\infty} \frac{A^2}{2T} \int_{-T}^{+T} \left[\frac{1}{2}\cos(\omega\tau) - \frac{1}{2}\cos(2\omega t - \omega\tau + 2\epsilon)\right] dt \\ &= \frac{A^2}{2}\cos(\omega\tau) \end{aligned}$$

6 Aufgabe 11.32

$$\begin{aligned} E(k_z) &= \int_{-b/2}^{b/2} A_0 \cos(\pi z/b) e^{ik_z z} dz \\ &= A_0 \int \cos\frac{\pi z}{b} \cos k_z z dz + iA_0 \int \cos\frac{\pi z}{b} \sin k_z z dz \\ &= A_0 \cos\frac{bk_z}{2} \left[\frac{1}{(\frac{\pi}{b} - k_z)} + \frac{1}{(\frac{\pi}{b} + k_z)}\right] \end{aligned}$$