## Problem 1

- In the fully quantized model of a two-level atom interacting with a quantized field within the rotating wave approximation (think about the Jaynes–Cummings model), we have obtained the exact resonance solution for the initial state where the atom is excited and where the field is in a number state $|n\rangle$. Recall that the Rabi oscillations of the atomic inversion are periodic for this case. Use that solution to obtain the expectation value of the atomic dipole moment operator? Comment on the result.

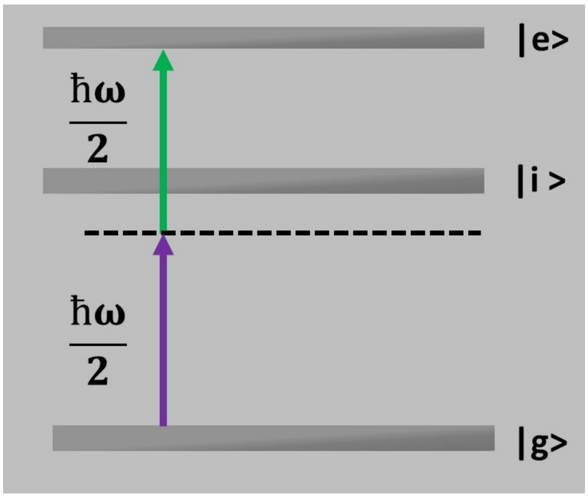
- Obtain the expectation value of the atomic dipole moment as given by the Jaynes–Cummings model in the case where the field is initially in a coherent state. How does the result compare with the two previous cases? You should make a plot of the expectation value of the dipole moment as a function of time.

## Problem 2

A resonant two-photon extension of the Jaynes–Cummings model is described by the effective Hamiltonian:

$$H_{eff} = \hbar\eta(\hat{a}^2\hat{\sigma}_+ + \hat{a}^{2\dagger}\hat{\sigma}_-)$$

where, for the sake of simplicity, a small Stark shift term has been ignored. This Hamiltonian represents two-photon absorption and emission between atomic levels of like parity. The process is represented by Fig.below, where the broken line represents a virtual intermediate state of opposite parity.




- Obtain the dressed states for this system.

- Obtain the atomic inversion for this model assuming the atom initially in the ground state and that the field is initially in a number state. Repeat for a coherent state. Comment on the nature of the collapse and revival phenomena for these states.

## Problem 3

Consider a two-level system initially in an excited state is placed in a cavity of quality factor $Q = \frac{\omega_0}{\kappa}$. The cavity supports only single mode. The interaction Hamiltonian is given by,

$$\hat{H}_1 = \hbar\lambda(\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger\hat{\sigma}_-)$$

and the master equation for the evolution of the density operator is given by,

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{H}_1, \hat{\rho}] - \frac{\kappa}{2}(\hat{a}^\dagger\hat{a}\hat{\rho} + \hat{\rho}\hat{a}^\dagger\hat{a}) + \kappa\hat{a}\hat{\rho}\hat{a}^\dagger$$

- Solve the master equation to determine the decay rate of the excited state.
- What will be the excited state decay rate if the quality factor of the cavity is very high such that $\frac{\omega_0}{Q} < 2\Omega_0$.
- What will be the excited state decay rate in the case of strong cavity damping $\frac{\omega_0}{Q} > 2\Omega_0$. Discuss the results.