Problem 1

An electron is in the spin state

$$\chi = A \begin{pmatrix} 3i \\ 4 \end{pmatrix}$$

- Determine the normalization constant A.
- Find the expectation value of S_x, S_y and S_z.
- Find the "uncertainities" σ_{S_x}, σ_{S_y} and σ_{S_z}.(Note: These sigmas are standard deviations, not Pauli matrices!).
- Confirm that your results are consistent with all three uncertainity principles.

Problem 2

An electron is in the spin state

$$\chi = A \begin{pmatrix} 1-2i \\ 2 \end{pmatrix}$$

- Determine the constant A by normalizing χ.
- If you measured S_z on this electron, what values could you get, and what is the probability of each? What is the expectation value of S_z?
- If you measured S_x on this electron, what values could you get, and what is the probability of each? What is the expectation value of S_x?
- If you measured S_y on this electron, what values could you get, and what is the probability of each? What is the expectation value of S_y?

Problem 3

Construct the matrix S_r representing the component of spin angular momentum along an arbitrary direction $\hat{r}$. Use spherical coordinates, for which:

$$\hat{r} = \sin\theta\cos\phi\hat{i} + \sin\theta\sin\phi\hat{j} + \cos\theta\hat{k}.$$

Find the eigenvalues and (normalized) eigenspinors of S_r.

Problem 4

An electron is at rest in an oscillating magnetic field

$$B = B_0\cos(\omega\ t)\hat{k},$$

Where B_0 and ω are constants.

- Construct the Hamiltonian matrix for this system.

- The electron starts out (at t $= 0$) in the spin-up state with respect to the x-axis (that is $\chi(0) = \chi_+^{(x)}$). Determine $\chi(t)$ at any subsequent time. Beware: This is a time-dependent Hamiltonian, so you cannot get $\chi(t)$ in the usual way from stationary states. Fortunately, in this case you can solve the time dependent Schrödinger equation directly.

- What is the minimum field (B_0) required to force a complete flip in S_x?

Problem 5

Suppose two spin $-1/2$ particles are known to be in the singlet configuration. Let $S_a^{(1)}$ be the component of the spin angular momentum of particle number 1 in the direction defined by the unit vector $\hat{a}$. Similarly, let $S_b^{(2)}$ be the component of 2's angular momentum in the direction $\hat{b}$. Show that:

$$< S_a^{(1)} S_b^{(2)} >= -\frac{\hbar^2}{4} \cos\theta,$$

where θ is the angle between $\hat{a}$ and $\hat{b}$.