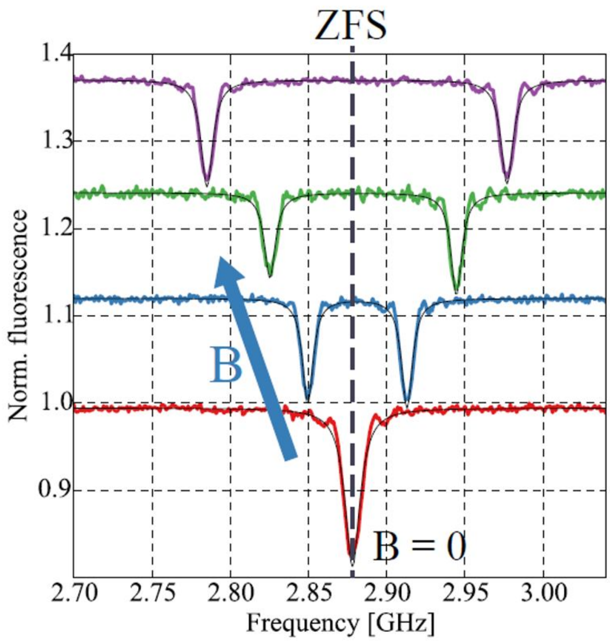
Problem 1

Negatively charged nitrogen-vacancy color centers in diamond has a ground-state triplet (^3A), and an excited-state triplet (^3E) along with SO coupled two intermediate-singlet states (^1A and ^1E).

1. Use the energy level diagram to discuss its spin state dependent fluorescence and optical polarization for quantum state preparation. What are the important consequences?
2. Typical ODMR spectra are recorded by sweeping the frequency of the microwave to drive the system from $m_S = 0 \leftrightarrow m_S = \pm 1$ transition and record the corresponding response by observing the change in the fluorescence of the color centers, as shown below. Discuss the approach in detail and derive an expression to find the magnitude of unknown magnetic field B.



Problem 2

You want to measure a weak magnetic field $B_{unknown}$ of unknown quantum system (it can be a spin) using quantum coherence that involves Ramsey sequences, Ramsey-type pulsed magnetometry, as depicted in figure below. A bias magnetic field $B_0\hat{z}$ along the NV-symmetry axis Zeeman-splits $m_s = \pm 1$ ground state allowing a two-level subspace that can be used as a basis to describe the Hamiltonian to study the dynamics.

1. What is the Hamiltonian of the system before the application $\frac{\pi}{2}$-pulses.
2. The application of oscillating magnetic field $\vec{B_1}(t) = B_1 \cos(\omega\ t)\hat{y}$ perpendicular to the NV-symmetry axis perturb the system. Determine the Hamiltonian driven by this oscillating field? Use the interaction picture to find the state vector as a function of time.

3. Suppose the oscillating field is turned off abruptly after a duration of $\tau\frac{\pi}{2} = \frac{\pi}{2\Omega} = \frac{\pi}{\gamma_e B_1}$. What will be the (interaction) Hamiltonian and the evolution of the state vector, remind that the weak magnetic field of the unknown source perturbs the system.

4. Another oscillating field $B_2(t)$ is chosen to be along the xy plane at an angle θ with respect to the polarization direction of the first $\frac{\pi}{2}$-pulse $B_1(t)$. Find the transformed Hamiltonian and the corresponding time evolution of the state vector.

5. Use the information from (1) to (4) to determine the magnitude of the unknown magnetic field.

6. What is the effect of the integration time in the signal-to-noise ratio of the measured magnetic field? I suppose the contrast degrades with increasing integration time due to dephasing, decoherence, and spin lattice interaction. Is there any technique to recover the coherence and rephase?