Nanoscale Quantum -Optics,
SS 2021

# Exercise II

Abgabe: 28.04.21

## Problem 1

- Show that the visibility of the Hong-Ou-Mandel (HOM) interference between two photons is just the lower limit of indistinguishability.
- Show that visibility is the same as the indistinguishability when the two photons are in pure state.

The two photons are indistinguishable if their density matrices are equal i.e.

$$\hat{\rho_1} = \hat{\rho_2}$$

The indistinguishability is identified by,

$$J(\hat{\rho_1}, \hat{\rho_2}) = 1 - \frac{1}{2}||\hat{\rho_1} - \hat{\rho_2}||^2$$

where, $||\hat{\rho_1} - \hat{\rho_2}||^2$ is the operational distance between $\hat{\rho_1}$ and $\hat{\rho_2}$.
The operational distance has a maximum value of two and has a minimum value of zero. It is zero when $\hat{\rho_1}$ and $\hat{\rho_2}$ are perfectly distinguishable.
In the Hong-Ou-Mandel (HOM) interference setup, the visibility is defined as:

$$V_{HOM} = \mathrm{Tr}\{\hat{\rho_1}, \hat{\rho_2}\} = \frac{\mathrm{Tr}\{\hat{\rho_1}^2\} + \mathrm{Tr}\{\hat{\rho_2}^2\} - ||\hat{\rho_1} - \hat{\rho_2}||^2}{2}$$

Since $\mathrm{Tr}\{\hat{\rho_1}^2\} \leq 1$ and $\mathrm{Tr}\{\hat{\rho_2}^2 \leq 1\}$, one can see that,

$$V_{HOM} \leq J(\hat{\rho_1}, \hat{\rho_2})$$

Hence visibility is the lower limit to indistinguishability.
When the two input beams are exactly in pure state defined by $|\psi_1\rangle$ and $|\psi_2\rangle$, then:

$$\mathrm{Tr}\{\hat{\rho_1}^2\} = \mathrm{Tr}\{\hat{\rho_2}^2\} = 1$$

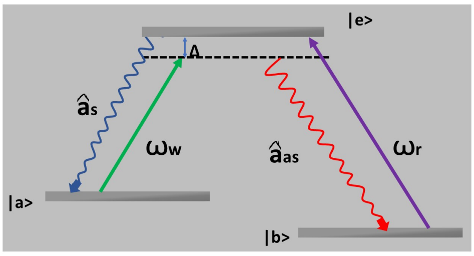
Hence,

$$V_{HOM} = 1 - \frac{1}{2}||\hat{\rho_1} - \hat{\rho_2}||^2 = J$$

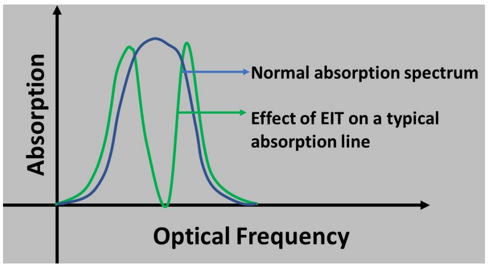
Which implies that the visibility and the indistinguishability are equivalent.

## Problem 2

- Describe the working principle of Write-Read based Heralded deterministic single-photon sources (use $\Lambda$ - type three level atomic system).
- Can you make the schematics of the experimental setup.
- How the Stoke photons can propagate through the atomic ensemble with little or no absorption.



(a)



(b)

For the generation of Heralded single-photon sources, first all atoms are initially prepared in the ground state $|a\rangle$ as shown in the $\Lambda$-type three level atomic system in fig(a). This can be possible using optical pumping. Then basic idea involves the use of **Write and Read process** as discussed below.

**Write process**:

In the first process off-resonant laser pulse with frequency $\omega_m$ couple the state $|e\rangle$ and $|a\rangle$. The laser pulse is applied to the atomic ensemble. Remind that the involved states are immune to the spontaneous emission. The atoms then decay in the state $|b\rangle$ by emitting **anti-stokes photons,** $\hat{a_{as}}$ (spontaneous Raman scattering). Energy conservation dectates that the number of atoms transferred to state $|b\rangle$ is equal to the number of anti-stokes photons.

**Read process**:

An on-resonant pulse is applied to convert the collective excitation to **stokes photons,** $\hat{a_s}$. Remind that weak write pulse creates only one anti-stokes in the mode of interest, then only one atom changes its state, but, it is difficult to know which one even in principle. The excitation rate must be low so that the probability that more than one photon is created is negligible. The collective excitation can be converted in to a single photon in the read process.

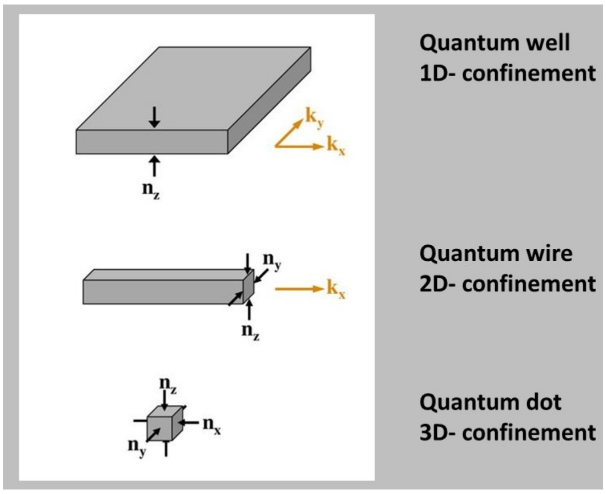
The stokes photon and the strong read light pulse satisfy the electromagnetically induced transparency (EIT) condition, hence the stokes photon can propagate through the atomic ensemble with littile or no absorption. In the ideal case, the excitation stored in the atomic ensemble can be deterministically retrieved. Electromagnetically Induced Transparency (EIT) is an important coherent optical quantum interference which creates a narrow transparency spectral window around an absorption line as shown in fig(b). If you are curious to understand EIT, answer the questions in the appendix.

## Problem 3

- Describe the physics behind the size dependent optical and electronic properties of semiconductor nanoparticles used for single-photon sources.
- What are quantum well, quantum wire and quantum dots.

The excitation in a semiconductor is due to the creation of an electron-hole pair, called **excitons**. When the size of the semiconductor is small, nearly the same as the spatial extent of the excitonic wavefunction, the quantum confinement effects becomes visible and one can see size and shape dependent emission properties like particle in a box model.

This confinement descretize the energy level and the nanocrystals acts like artificial atoms. Hence, can be used as photon sources.



(c)

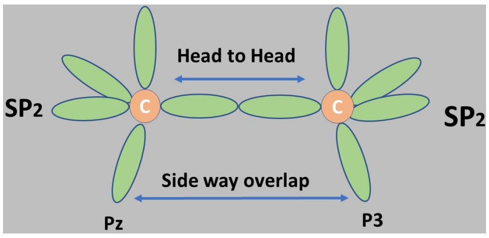
Quantum confinement in nanocrystals can be 1D-quantization for example quantumwell, where the excitons are confined in 1-dimension, while it is free to move in 2-dimensions. Similarly, 2D-quantization in quantum wire leads to 2-dimensions of excitonic confinement. While in the case of 3D-quantization (quantum dot), the excitons are confined in all 3-dimensions. (see fig(c)).

## Problem 4

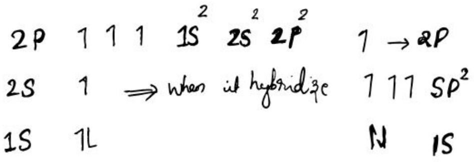
- Explain how $\pi$- conjugated molecules ($SP_2$-hybridized orbitals) leads to quantum confinement of delocalized electronics and used as a single-photon sources.
- Is it possible to tune their emission properties, if so explain how ?. Consider the following fluorescence dye molecule.

Carbon atom has 6 electron and $\pi$ conjugated molecules undergo $SP_2$-hybridization of orbitals. The hybrid orbitals form a triangonal planar geometry. The unhybridized $2P_z$ orbitals are perpendicular to the plane and overlap side-by-side to form a weaker $\pi$ bond as shown in the fig(d).

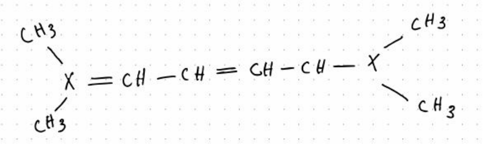
If the molecules has an alternation of double and single bonds over a planar segment, the system is $\pi$-conjugated. The overlap between $\pi$-orbitals in the structure allows the delocalization of the electron over the whole conjugate segments. The movement of delocalized electron is only restricted due to the repulsive potential of the methyl group at the end of the chain as shown in fig(f). This allows to trap the electron in a potential well and hence discretization of energy levels and possible photon emission. Moreover, by changing the length of the bond (double and single bonds), the color of the luminescence molecule can be tuned.



(d)

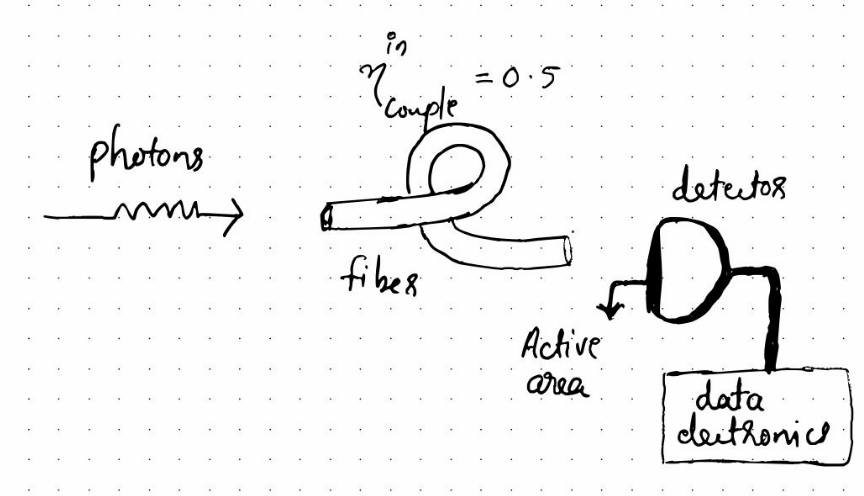


(e)



(f)

## Problem 5



- Silicon-vacancy based single-photon source, emission wavelength at 738 nm, emits photons at a rate of $10^6 s^{-1}$. The photons are detected using a fiber coupled detector. The parameters associated with the detector and those associated with the experiment is given below. Determine the detection efficiency ?

  The fiber transmission $\eta_{fiber} = 0.6$ at 738 nm, the outcoupling efficiency is $\eta^{out}_{coupling} = 0.4$. The absorption efficiency $\eta_{absor}$ of the detector is 0.5. The internal quantum efficiency $\eta_{QE}$, that measure the fraction of absorbed photons that yield an output electric signal is 0.4. The readout electronics threshold efficiency, $\eta_{thresh}$, qunatifies the efficiency with which the output electronic signal is represented by external counting or timing electronic is 0.95.

$$\eta = \eta_{fiber} \times \eta_{coupling} \times \eta_{absor} \times \eta_{QE} \times \eta_{thresh}$$

$$\eta = 0.6 \times 0.4 \times 0.5 \times 0.4 \times 0.95 = 0.0456 = 4.5\%$$

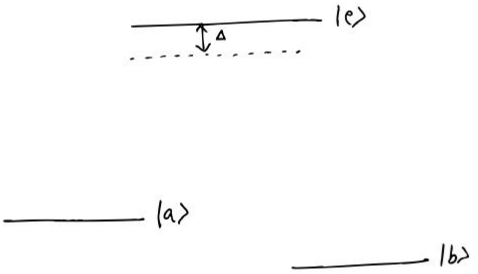
- What will be the detection efficiency if we use free-space coupling.

$$\eta_{freespace} = \eta_{absor} \times \eta_{QE} \times \eta_{thresh}$$

$$\eta_{freespace} = 0.5 \times 0.4 = 0.2 = 20\%$$

## Appendix

- What is effect of the coherent laser on the absorption and dispersion?
  The polarizability can be calculated by looking theinduced dipolemoment.



- Calculate the induced dipole-moment at the probe frequency (depending on the off-diagonal element of the dipole matrix)