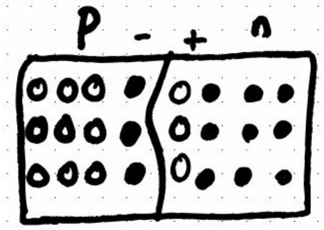
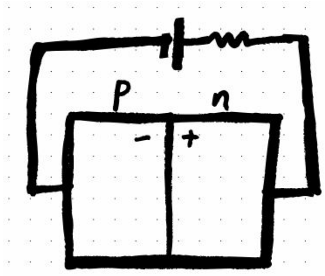
Problem 1

An avalanche photodiode contains p^+ip and n^+ region as shown below. The photodiode is reverse biased. The reverse bias voltage increases the potential difference across the depletion region (remember the polarity in the depletion region is as shown below).





In a typical junction (e.g. pn-junction) the field developed in the depletion region is enough to stop carrier diffusion. The assiciated barrier voltage is also called buit-in voltage as shown in figure below. The reverse bias widened the depletion region and the field becomes stronger.

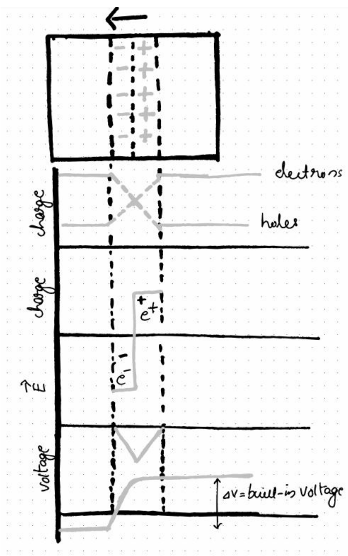


Figure 1: Built-in voltage due to pn-junction

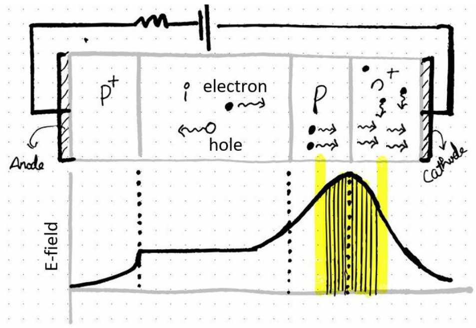


Figure 2: Electricfield along the different regions of the junction

The photons reaching the i-region generate electron and hole through photoelectric effect. The created electrons and holes are rapidly seperated and swept out of the depletion region. The generated electrons can generate multiple electrons through impact ionization in the pn_+ region as the elctricfield is extremely high.

Typical APDs show an internal current gain effect around 100 due to impact ionization, some more than > 1000. In general the higher the reverse voltage, the higher the gain.

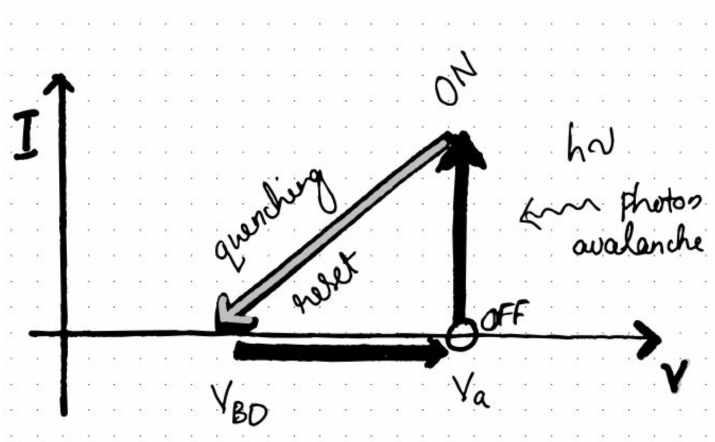
For single-photon detection 10^5 to 10^6 gain is needed and the photodiode should operate at very high bias voltage. The responsivity (sensitivity) is related to the multiplication factor according to,

$$I_P = q \times N_e^M$$

where N_e is the number of carrier generated and M is the multiplication factor.

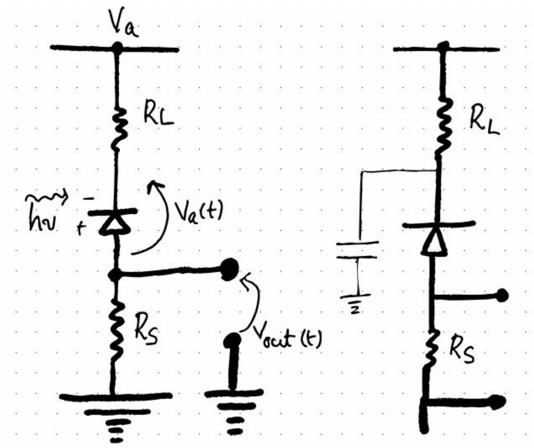
This current is due to the impact ionization (not by reverse bias) and this relation shows that APDs are not linear devices.

Consider the APD operation in the IV curve shown below.



Photon assisted carrier generation trigger the avalanche effect and switch the device to ON state. The device stays ON until the avalanche is quenched by an external circuit (quenching circuit) and reset for the other photon event detection.

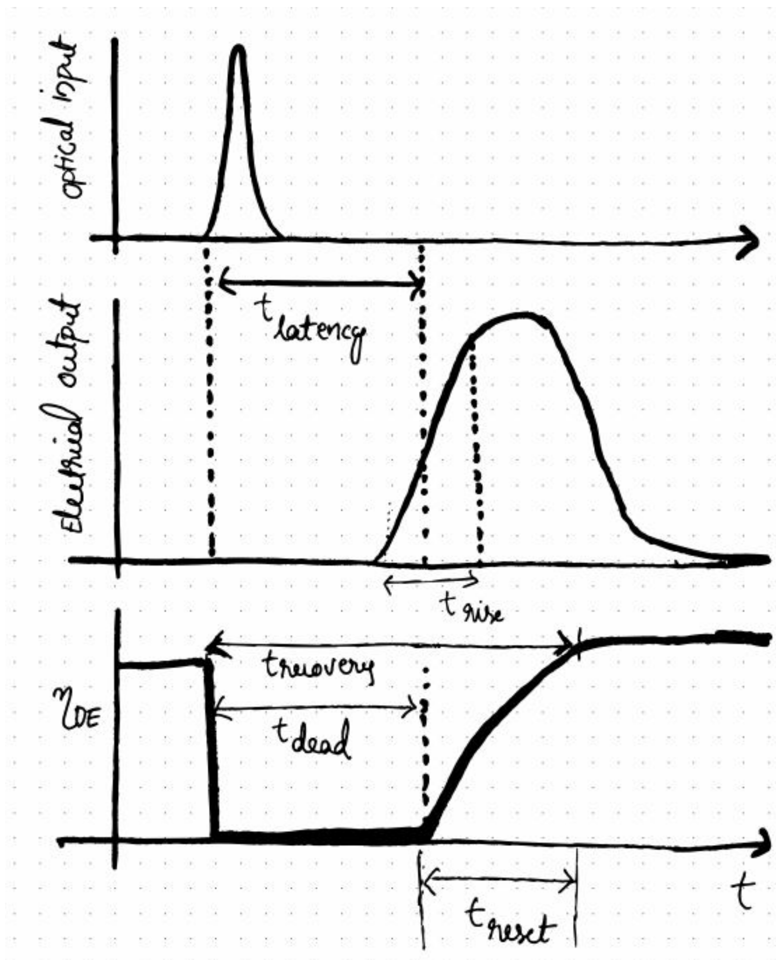
Passive-quenching circuit : The circuit that quenches the avalanche and the bias voltage plays an integral role. Consider the following circuit diagram:



The bias voltage is applied through a large ballast resistor R_L. A small resistor R_S is connected to the other terminal for observing the current pulse. The avalanche current discharges the total capacitance at the diode terminal. The voltage across the diode decreases toward V_{BD} and the avalanche decreases. As the voltage decreases to V_{BD}, the rate of avalanche decreases. All the avalanche current flows through R_L and is reduced to, $\frac{V_a - V_{BD}}{R_L}$.

The current flowing through the APD is limited by R_L. This current will quickly discharge the junction capacitor until the voltage drops below V_{BD}. The time constant of effective R_C circuit is $\tau \approx R_L \times C \approx$ dead time.

The voltage that starts to recover slowly towards the bias voltage V_a (reset transition), as small current in R_L recharges the capacitor with long time constant. The ON-OFF-Reset process leads to a time interval in which the device is ready to detect other photons. The tpical property is shown below.



Problem 2

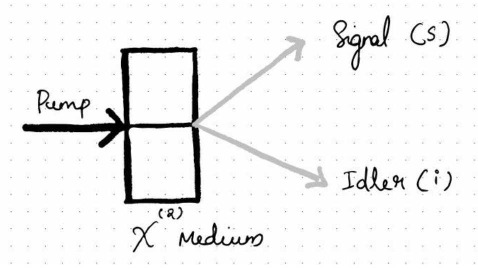
The calibration based on the spontaneous parametric down conversion involves the use of a second order non-linear process ($\chi^{(2)} \neq 0$) for the generation of photon pairs.

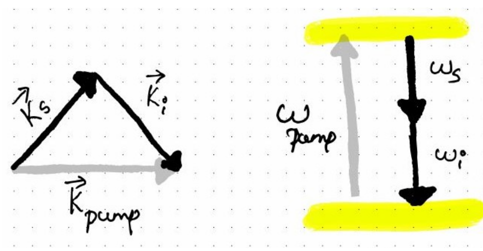
$$\vec{P}(t) = \epsilon_0 \Big(\chi^{(1)} \vec{E}(t) + \chi^{(2)} \vec{E}^2(t) + ... \Big)$$

where $\chi^{(n)}$, is the n^{th} oder susceptibility of the medium, and $\vec{P}(t)$ is the polarization density (electric dipole moment per unit volume).

The process satisfies momentum and energy conservation. As shown in the setup, the detection efficiency of the detector under test (DUT) is the raio of the number of coincidence events to the number of trigger detection events in a given time interval.

Coincidences are usually measured with start-stop time-to-digital system, with the heralding detector output connected to the start and DUT output to the stop.

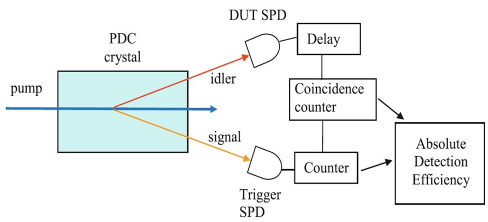


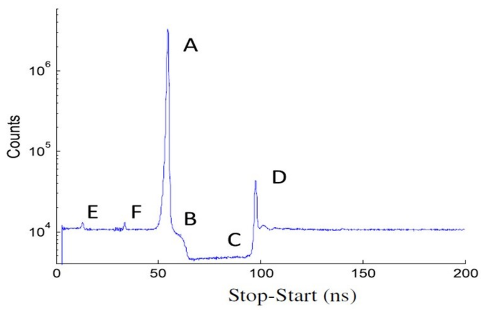


If N_P is the total number of down converted photons emitted in to the trigger channel during the counting period.
The number of trigger $N_{trig} = \eta_{trig} \times N_P$.
Where η_{trig} is the total efficiency of trigger channel.
Then the total number of coincidences count is given as:

$$N_c = \eta_{DUT} \times \eta_{trig} \times N_P$$

$$\eta_{DUT} = \frac{N_c}{N_{trig}}$$





Figure(b) in the question shows that

- A - The main correlation peak.
- B - Correlated photons during the reset mode of the detector exhibiting longer latency times.
- C - Reduction of stop counts due to the recovery time of a detector
- D - Peak due to after pulse and delay count in the reset mode (twilight counts)
- E and F - Specific to setup not to DUT.

Problem 3

We know that $\hat{H} = \hat{H}_0 - \hat{d}.\vec{E}(t)$, where $\hat{d} = -e\vec{r}$.

Consider the case in which the atom is initially in the state $|i\rangle$ and its time evolution is:

$$|\psi(t)\rangle = e^{\frac{-i\hat{H}t}{\hbar}} |i\rangle$$

Expand interms of complete set of uncoupled atomic states $|k\rangle$

$$\sum_k |k\rangle \langle k| = \hat{I}$$

$$|\psi(t)\rangle = e^{\frac{-i\hat{H}t}{\hbar}} \hat{I} |i\rangle = \sum_k e^{\frac{-i\hat{H}t}{\hbar}} |k\rangle \langle k|i\rangle$$

$$|\psi(t)\rangle = \sum_k e^{\frac{-iE_k t}{\hbar}} C_k |k\rangle$$

or

$$\langle\psi(t)| = \sum_m e^{\frac{+iE_m t}{\hbar}} C_m \langle m|$$

Since $\langle\psi(t)|\psi(t)\rangle = 1$. Thus we have,

$$\sum_{mk} C_k C_m e^{\frac{+i(E_m - E_k)}{\hbar}} \langle m|k\rangle = 1$$

but $\langle m|k\rangle = \delta_{mk}$, this leads to:

$$\sum_k |C_k|^2 = 1$$

From time-dependent Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = (\hat{H}_0 + \hat{H}^I) |\psi(t)\rangle$$

but,

$$|\psi(t)\rangle = \sum_k C_k(t)\ e^{\frac{-iE_k t}{\hbar}} |k\rangle$$

now multiply by $\langle l| e^{\frac{iE_l t}{\hbar}}$,

$$i\hbar \frac{\partial}{\partial t} \langle l| e^{\frac{iE_l t}{\hbar}} |\psi(t)\rangle = \langle l| e^{\frac{iE_l t}{\hbar}} \hat{H}_0 |\psi(t)\rangle + \langle l| e^{\frac{iE_l t}{\hbar}} \hat{H}^I |\psi(t)\rangle$$

$$i\hbar \frac{\partial}{\partial t} C_l(t) = \sum_k C_k(t) \langle l| \hat{H}^I |k\rangle e^{\frac{i(E_l - E_k)}{\hbar}}$$

or

$$\frac{\partial}{\partial t} C_l = -\frac{i}{\hbar} \sum_k C_k \langle l| \hat{H}^I |k\rangle e^{i\omega_{lk} t}$$

where $\omega_{lk} = \frac{E_l - E_k}{\hbar}$.

If $C_i(0) = 1$, implies that only state $|i\rangle$ is initially populated, later $|f\rangle$ has a definite probability to be populated.

$$P_{i\to f}(t) = C_f^*(t)C_f(t)$$

Which indicates the probablility for the atom to make a transition from $|i\rangle$ to $|f\rangle$ in time t.

So,

$$\dot{C}_l(t) = -\frac{i}{\hbar}\sum_k C_k \langle l| \hat{H}^I |k\rangle e^{i\omega_{lk}t}$$

Where, $\hat{H}^I = -\hat{d}.\vec{E}_0 \cos\omega t$ but $\cos\omega t = \frac{e^{i\omega t}+e^{-i\omega t}}{2}$. Near resonance $\omega \approx \omega_{fi}$ using rotating wave approximation and by neglecting rapidly oscillating terms:

$$P_{i\to f}(t) = |C_f(t)|^2 = \frac{|(\hat{d}.\vec{E}_0)_{fi}|^2}{\hbar^2} \times \frac{\sin^2(\frac{\Delta t}{2})}{\Delta^2}$$

where, $\Delta = \omega - \omega_{fi}$ is the detuning.

$$(\hat{d}.\vec{E}_0)_{fi} = \langle f| \hat{d}.\vec{E}_0 |i\rangle$$

When $\vec{E}_0 = 0 \implies P_{i\to f}(t) = 0$.

This contradicts the possibility of observing spontaneous emission if the atom is in initially in the higher energy level. That is why spontaneous emission is purely quantum mechanical effect (require quantization of the field).

Problem 4

The Hamiltonian is given by,

$$\hat{H} = \hat{H}_0 - \vec{d}.\vec{E}$$

but,

$$\hat{E} = i\Big(\frac{\hbar\omega}{2\epsilon_0 V}\Big)^{1/2}\vec{e}\,[\hat{a} - \hat{a}^\dagger]$$

$$\hat{H}^I = -\vec{d}.\vec{E} = -i\Big(\frac{\hbar\omega}{2\epsilon_0 V}\Big)^{1/2}(\hat{d}.\vec{e}) \quad [\hat{a} - \hat{a}^\dagger]$$

$$\hat{H}^I = -\vec{d}.\vec{\epsilon_0}[\hat{a} - \hat{a}^\dagger]$$

Here, $\vec{\epsilon_0} = i\Big(\frac{\hbar\omega}{2\epsilon_0 V}\Big)^{1/2}\vec{e}$.

The initial state of the atom-field is,

$$|i\rangle = |a\rangle\,|n\rangle$$

This atom state transforms into one of the following final state.
For absorption,

$$|f_1\rangle = |b\rangle\,|n-1\rangle$$

For emission,

$$|f_2\rangle = |b\rangle\,|n+1\rangle$$

The matrix element of the interaction is,

$$\langle f_1|\,\hat{H}^I\,|i\rangle = \langle b, n-1|\,\hat{H}^I\,|a,n\rangle = -(\hat{d}.\vec{E_0})_{ba}\sqrt{n} \implies (absorption)$$

similarly,

$$\langle f_2|\,\hat{H}^I\,|i\rangle = \langle b, n+1|\,\hat{H}^I\,|a,n\rangle = (\hat{d}.\vec{\epsilon_0})_{ba}\sqrt{n+1} \implies (emission)$$

When there are no photons, n = 0.
The emission,

$$\langle f_2|\,\hat{H}^I\,|i\rangle = \langle b, n+1|\,\hat{H}^I\,|a,n\rangle = (\hat{d}.\vec{\epsilon_0})_{ba}\sqrt{n+1} = (\hat{d}.\vec{\epsilon_0})_{ba} \implies (spontaneous\ emission)$$

Uisng Schrödinger equation, the state vector can be written as:

$$|\psi(t)\rangle = C_i(t)\,|a\rangle\,|n\rangle\,e^{\frac{-iE_at}{\hbar}}\;e^{-in\omega t} + C_{f1}(t)\,|b\rangle\,|n-1\rangle\,e^{\frac{-iE_bt}{\hbar}}\;e^{-i(n-1)\omega t} + C_{f2}(t)\,|b\rangle\,|n+1\rangle\,e^{\frac{-iE_bt}{\hbar}}\;e^{-i(n+1)\omega t}$$

We assume that $|\psi(0)\rangle = |a\rangle\,|n\rangle\,, C_i(0) = 1$ and $C_{f1}(0) = C_{f2}(0) = 0$.