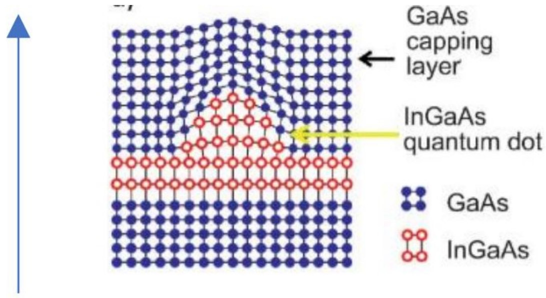
Problem 1

Typical quantum dots used for spin manipulation has strong confinement of excitons along the growth direction that can be modelled as an infinitely deep potential box. The symmetry of the in-plane confinement allows for qualitative description via a two-dimensional harmonic oscillator. Consider for example InGaAs/GaAs quantum dot shown below of size 30-40 nm wide and a height of 5 nm along the growth direction.



- Find the wave function of the excitons and its energy eigenvalues.
- The application of light with frequency ω creates excitons if it satisfies the spin selection rule. Let us consider the excitation light is right circularly polarized, what conditions leads to the creation of bright and dark excitons? Can you write the state of the system using spin states? Remind that electrons in the conduction band has s-like symmetry and heavy holes in the valence band have p-like symmetry.

The strong confinement along the growth direction can be modelled as an infinitely deep potential bix. The symmetry of the in-plane confinement allows for qualitative description via a two dimensional harmonic oscillator.

$$\left[-\frac{\hbar^2}{2m^*}\nabla^2 + V(r,z)\right]\psi_n(r,z) = E_n\psi_n(r,z)$$

$$V(r,z) = \left\{\frac{1}{2}m^*\omega_0^2\, r^2, \quad \text{for} \quad |Z| \leq \frac{d}{2}\right\}$$
$$= \left\{\infty, \quad \text{for} \quad |Z| > \frac{d}{2}\right\}$$

The wave function can be seperated using the product rule,

$$\psi_n(r,z) = \phi_{nr}(r) \quad \chi_{nz}(z)$$

Then,

$$-\frac{\hbar^2}{2m^*}\nabla^2\phi_{nr}(r) + \frac{1}{2}m^*\omega_0^2\, r^2\, \phi_{nr}(r) = E_{nr}\phi_{nr}(r)$$

The solution for these differential equation is given by:

$$\phi_{nr}(r) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n n!}} \quad H_n(r) \quad e^{\left[-\frac{\omega_0 m^*}{2\hbar} r^2\right]}$$

with energy eigenvalue, $E_n = (n + 1/2)\hbar\omega, n = 0, 1, 2.....$

The strong confinement along the growth direction is given as : as $V = \infty$,

$$-\frac{\hbar^2}{2m}\nabla^2\chi(z) = E_{nz} \quad \chi(z)$$
$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} \quad \chi = E_{nz} \quad \chi$$

has a solution of the form:

$$\chi(z) = A\sin kz + B\cos kz$$

Boundary conditions dictates:

$$\chi(z) = A\sin\left(\frac{\pi n_z \ z}{d} + \frac{\pi n_z}{2}\right), \quad \text{with} \quad E_{nz} = \frac{n^2\pi^2\hbar^2}{2md^2}$$

Hence,

$$|\psi_n\rangle \propto H_n(r) \quad e^{\left[-\frac{\omega_0 m^*}{2\hbar} r^2\right]} \quad \sin\left(\frac{\pi n_z \ z}{d} + \frac{\pi n_z}{2}\right), \quad \text{where} \quad n_{rz} = 1, 2, ...$$

$$E_n = \hbar \ \omega_0 \ n_r + \frac{\hbar^2 \ \pi^2 (n_z)^2}{2m^* d}, \text{where} \quad n_{rz} = 1, 2, ...$$

Electrons in the ocnduction band have S-like symmetry of the Bloch wave function implying an angular momentum of $J = \pm 1/2$. Let's represent the states by $\uparrow$ or $\downarrow$. Heavy holes have P-like symmetry with $J = \pm 3/2$. Let's represent the states by $\Uparrow$ or $\Downarrow$.

If the conduction band is unocuupied, the system has a total momentum of $J = 0$. We know that right or left circularly polarized light (photon) has angular momentum of $J = 1$.

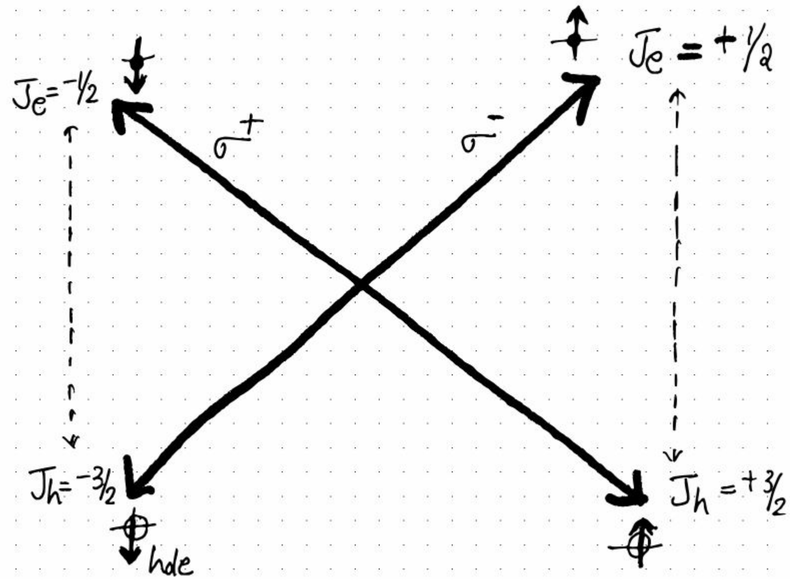
Depending on the helicity of the photon $J = \pm 1$ for σ^+ or σ^- polarization only transitions with $\Delta J = \pm 1$ are optically allowed.

Hence,

$$|cgs\rangle \xrightarrow{\sigma^+} |\downarrow\Uparrow\rangle \quad \text{or} \quad |cgs\rangle \xrightarrow{\sigma^-} |\uparrow\Downarrow\rangle$$

here, cgs is the crystal ground state.

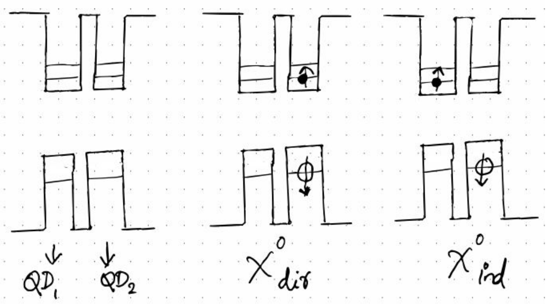
Neutral excitons with a total momentum of $J = \pm 1$ are term <u>bright</u>. Excitons with $J = \pm 2$ ($|\downarrow\Downarrow\rangle, |\uparrow\Uparrow\rangle$) are dipole forbidden, hence called <u>dark excitons</u>.



Problem 2

Consider two dots (quantum dot molecule) that are only separated by a thin barrier, e.g., 3 nm of GaAs between InGaAs dots, and the interactions between states of the individual dots significantly depend on their relative energy detuning resulting in the formation of a coupled system. Show that this effect leads to an avoided crossing behavior, 'anticrossing', as two interacting states are tuned in and out of resonance.

For the neutral exciton state χ^0, electron and hole can be located in one of the two dots (direct) or spatially seperated confined in different dots od the molecule (indirect).



Since the dots are only seperated by a thin barrier (3 nm of GaAs between InGaAs) the interaction between states of the individual dots can become significantly strong.

At first $H^0\psi_m = E_m\ \psi_m$, where m = direct, indirect. If the two states are interacting via a coupling κ, the new eigenstates of the system can be written as linear combinations.

$$\phi_i = \alpha\ \psi_{dir} + \beta\ \psi_{ind}$$

or

$$|\phi_i\rangle = \alpha\ |\psi_{dir}\rangle + \beta\ |\psi_{ind}\rangle$$

and , $|\alpha|^2 + |\beta|^2 = 1$.

The coupled Hamiltonian H_c can be written as $\hat{H}_c = \hat{H}_0 + \hat{\kappa}$ (First order perturbation theory).

Using basis $|\psi_{dir}\rangle$ and $|\psi_{ind}\rangle$, one can write the Hamiltonian as:

$$\begin{pmatrix} \langle\psi_{dir}|\hat{H}_c|\psi_{dir}\rangle & \langle\psi_{dir}|\hat{H}_c|\psi_{ind}\rangle \\ \langle\psi_{ind}|\hat{H}_c|\psi_{dir}\rangle & \langle\psi_{ind}|\hat{H}_c|\psi_{ind}\rangle \end{pmatrix}$$

Which implies that:

$$\boxed{H_c = \begin{pmatrix} E_{dir} & \kappa_{\frac{dir-ind}{2}} \\ \kappa^*_{\frac{ind-dir}{2}} & E_{ind} \end{pmatrix}}$$

Where, $\kappa_{dir-ind} = \kappa^*_{ind-dir} = \langle\psi_{dir}|\hat{\kappa}|\psi_{ind}\rangle$, here $\kappa_{dir-ind}$ is the coupling matrix element.

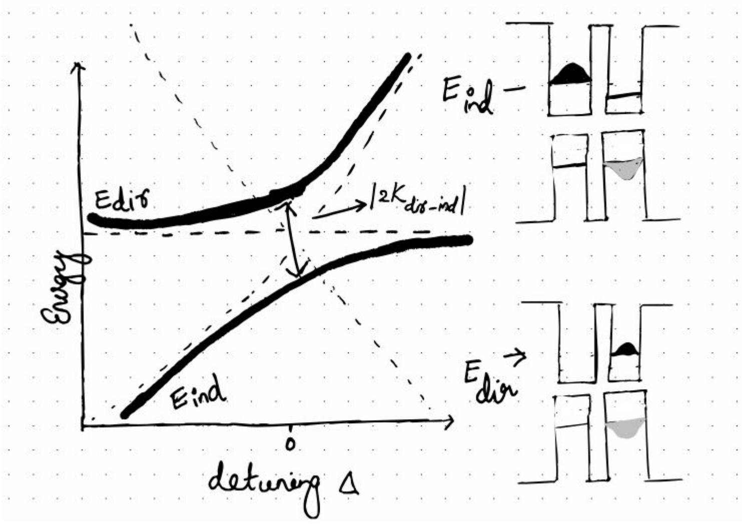
The eigenvalues of the coupled system H_c can be expressed interms of the detuning between the uncoupled states.

$$\Delta_{dir-ind} = E_{dir} - E_{ind}$$

$$\boxed{E_\pm = E_{dir} + \frac{1}{2}\Big(\Delta_{dir-ind} \pm \sqrt{\Delta^2_{dir-ind} + \kappa^2_{dir-ind}}\Big)}$$

The eigenstates are symmetric and antisymmetric linear combinations of the uncoupled states ψ_{dir} and ψ_{ind} forming bonding (ϕ_+) and antibonding (ϕ_-) molecular states.

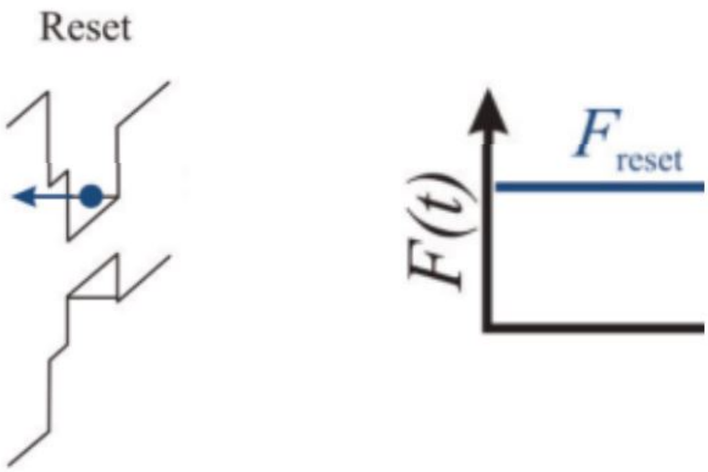
$$\phi_+ = \frac{1}{\sqrt{2}}(\psi_{dir} + \psi_{ind})$$
$$\phi_- = \frac{1}{\sqrt{2}}(\psi_{dir} - \psi_{ind})$$



At zero detuning between the states when the electron energy levels of both dots are aligned, resonant tunneling process between the upper and lower quantum dots take place forming a coupled system with eigenenergies E_+ and E_-.

Problem 3

One of the techniques used for optical manipulation and read out of spins in quantum dots (e.g., InGaAs embedded in a GaAs photodiode structure) is to use operations that involve the application of electric field and lasers in the following order:
reset. . . >charging. . . .>Storgae. . . ..> spin-to-charge conversion. . . .>readout.



1. Can you show the energy diagram and discuss the operation to create, manipulate and read the spin state of an electron?
2. During the reset process discharging of the quantum dots is needed as shown below using appropriate electric field F to create spin up or down state. What do you do if you want to create spin up state by optical excitation? What polarization of light should be used? Discuss the techniques that will allow you to read the spin state after some storage time.
3. Interaction with the external environment may flip the spin state discussed in (2). How do you know if the spin is flipped?

One of the technique to optically create and sense a single electron spin in a quantum dot with a predefined spin polarization is by using the voltage tunable spin memory device (see figure below to follow the discussion).

We start with a reset operation by the application of strong electric field (F_{reset}) inorder to empty the quantum dot from residual charges. In the next step, switch the electric field strength to a lower value (F_{ch1}) and apply a laser.

$$|cgs\rangle \rightarrow |\chi^{\circ}\rangle\,, \quad \text{optical} \quad \text{transition}$$

Hence, circularly polarized pump pulse resonantly creates exciton with an electron in the spin up or spin down with respect to the optical axis. The hole will be tunnels out of the quantum dot as a result of the applied field leaving behind a single electron. The electric field is chosen such that the hole removal time is much faster than the time scale for exciton fine structure precession providing a high spin initialization fidelity. In addition, the electron tunneling is strongly suppressed by layers (Al Ga As barrier). Hence the electron can be stored for a controlled time T_{store} at selected electric fields F_{store} until the spin is tested.

Instead of direct probing the electron spin, spin-to-charge conversion can be performed before reading. Tune the electric field to an intermediate value F_{ch2} for hole tunneling and apply a second circularly polarized laser in resonance with $\boxed{\left|e^-\right\rangle \rightarrow \left|\chi^-\right\rangle}$ transition. The Pauli spin blockade either allow or inhibits light absorption as shown in the figure.

Finally, the device is biased into the charge readout mode by reducing the electric field to F_{read} where the optical recombination of the electron and hole dominates over tunneling. In order to read the charge occupancy of the quantum dot, apply additional laser pulse (read laser) energetically tuned in to resonance with $\boxed{\left|e^-\right\rangle \rightarrow \left|\chi_T^-\right\rangle^*}$ optical transition. The possible outcome is depicted in the figure below.