# 1 Relationship between dielectric constant and conductivity

Macroscopically, a metal can be considered as a material with a dielectric constant $\hat{\epsilon}_{\text{bound}}(\omega)$ due to bound electrons and a conductivity $\hat{\sigma}(\omega)$ due to free electrons.

1. Consider the Maxwell equation

$$\text{rot}\vec{B} = \mu_0\Big(\vec{j} + \frac{\partial \vec{D}_{\text{bound}}}{\partial t}\Big), \tag{1}$$

where $\vec{D}_{bound}$ is due to bound electrons and $\vec{j}$ is due to the free electrons, and in complex notation, $\hat{\vec{E}} = \vec{E}_0 e^{-i\omega t}$ is the electric field of a plane wave with frequency $\omega$. Using the known relation between the current density $\vec{j}$ and the electric field $\vec{E}$, show that

$$\text{rot}\hat{\vec{B}} = -i\frac{\omega}{c^2}\Big(\hat{\epsilon}_{\text{bound}} + i\frac{\sigma}{\epsilon_0\omega}\Big)\hat{\vec{E}}.$$

2. Consider now a metal as a dielectric with a dielectric constant $\hat{\epsilon}(\omega) = \hat{\epsilon}_{\text{bound}}(\omega) + \hat{\epsilon}_{\text{free}}(\omega)$, where $\hat{\epsilon}_{\text{free}}(\omega)$ is the contribution from free electrons. Using the Maxwell equation in eq.1, $\hat{\vec{D}} = \hat{\epsilon}\epsilon_0\hat{\vec{E}}$ and $\hat{\vec{j}} = 0$, relate the dielectric constant $\hat{\epsilon}_{free}(\omega)$ to the conductivity $\hat{\sigma}$.

# 2 Complex Refractive Index

The complex refractive index $\tilde{n}$ and complex dielectric constant $\tilde{\epsilon_r}$ are defined as

$$\tilde{n} = n + i\kappa$$
$$\tilde{\epsilon_r} = \epsilon_r' + i\epsilon_r''$$

Assuming the magnetic response $\tilde{\mu_r}$ of a material is negligible, the complex refractive index is related to the complex dielectric function through

$$\tilde{n}^2 = \tilde{\epsilon_r}$$

Derive the following equations for the real and imaginary parts of the complex refractive index in terms of the real and the imaginary parts of the dielectric constant. Show each step and justify the sign of the roots.

$$n = \sqrt{\frac{1}{2}(\sqrt{\epsilon_r'^2 + \epsilon_r''^2} + \epsilon_r')}$$
$$\kappa = \sqrt{\frac{1}{2}(\sqrt{\epsilon_r'^2 - \epsilon_r''^2} + \epsilon_r')}$$

# 3 Kramer-Krönig Relations and Cauchy Theorem

Using Cauchy's theorem, prove that the real and imaginary parts of the electric susceptibility derived from Lorentz Model satisfy the Kramers-Kronig relations.