Exercise 2

1 - Time-bandwidth product

Either numerically or analytically, construct a pulse whose time–bandwidth product in terms of FWHM duration and bandwidth is very small. Can you realize a $\Delta \nu \Delta t$ value of less than 0.1? Less than 0.01?

2 - Pulse characterisation through autocorrelation functions

Consider a frequency-modulated Gaussian pulse with center frequency $\omega_0$ and complex envelope function $a(t) = \exp(-t^2/t_p^2)\exp(j\phi(t)/t_c^2)$. Produce plots of the power spectrum, electric field autocorrelation envelope, (fringe-averaged) intensity auto-correlation, and interferometric intensity autocorrelation, for $t_p = 200$ fs, $\omega_0/2\pi = 5 \times 10^{14}$ Hz, and $t_c = 40, 200, \text{ and } 800$ fs. Comment on the trends in your plots. For the interferometric autocorrelation, it is sufficient to plot the envelopes of the fringes.

3 - Frequency resolved optical gating (FROG)

Demonstrate that spectrogram and sonogram time–frequency distributions, eqs. (3.70) and (3.71) of the book Ultrafast Optics, are identical, provided that the sonogram filter function and spectrogram gate function satisfy $H(\omega) = G(-\omega)$.

4 - Phase and group velocities

Examine the magnitude of $\lambda(d\nu/d\lambda)$ assuming that the refractive index is described by a single-term Sellmeier equation, written $n^2(\lambda) = 1 + A\lambda^2/(\lambda^2 - \lambda_0^2)$, where $\lambda_0 = 150$ nm and the refractive index is 1.44 at $\lambda = 1$ $\mu$m. Plot $\lambda(d\nu/d\lambda)$ throughout the visible wavelength range. What is the fractional difference between the group velocity and the phase velocity?

5 - Pulse propagation in a medium with quadratic (first-order) dispersion

Consider a chirped pulse propagating in a medium of length $L$ and dispersion $\beta_2$. The electric field envelope of the input pulse is given by $a_{in}(t) = \text{sech}(t/t_p)e^{j\phi(t)}$, where $\phi(t) = B \ln[\text{sech}(t/t_p)]$. Calculate and plot the intensity profile, temporal phase, and instantaneous frequency of the output pulse for $B = 10$, $t_p = 1$ ps, and $\beta_2L$ set to 0 or $\pm t_p^2/B$. Comment on your results.

Exercises selected from chapters 3 and 4 of Ultrafast Optics by A.M. Wiener (J. Wiley & Sons, 2009).